Modelling fuzzy quantified statements under a voting model interpretation of fuzzy sets

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Abstract. In this paper previous results by our group in the field of fuzzy quantification modelling and application are compiled. A general mechanism that is based on the voting-model interpretation of fuzzy sets is described. Application examples of quantified statements in the field of reasoning with fuzzy temporal rules, modelling of task-oriented vocabularies and information retrieval are presented.

Key words: Fuzzy quantification, semi-fuzzy quantifiers, theory of generalized quantifiers, extension of fuzzy operators, mobile robotics, information retrieval

1 Introduction

Modelling of fuzzy quantified statements is of great interest for tasks related with knowledge representation and reasoning, knowledge extraction, intelligent control, decision-making, fuzzy databases, information retrieval, etc. In spite of its importance for these and other fields, most of the approaches that are presented in the literature inherit the basic definitions of linguistic fuzzy quantifiers derived from the first approach [15], that solely represent a fuzzy quantity or proportion (absolute and relative quantifiers). Although this may be sufficient for some applications, this excludes dealing with more complex statements, such as those involving exception, comparative or non-quantitative quantifiers, therefore limiting expresiveness of fuzzy quantified statements. Furthermore, most of the approaches in literature fail to exhibit a plausible behaviour, since important properties for fuzzy quantification methods (correct generalisation, continuity, monotonicity, etc.) are not fulfiled [3, 12].

Following [11, 13] the evaluation of quantified sentences is presented under the approach of fuzzification of *semi-fuzzy quantifiers*. Under this approach the problem of evaluating a fuzzy quantified statement is rewritten as the problem

* Authors wish to acknowledge support from the Spanish Ministry of Education and Culture through grant TIC2000-0873. D. E. Losada is supported by the "Ramón y Cajal" R&D program, which is funded in part by "Ministerio de Ciencia y Tecnología" and in part by FEDER funds.

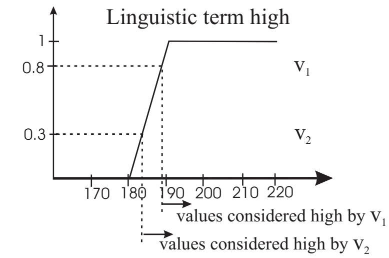

Fig. 1. Height values that are voted as "high" by two voters with different specificity.

of applying an adequate fuzzification mechanism for transforming a semi-fuzzy quantifier into a fuzzy quantifier. Some *quantifier fuzzification mechanisms* that secure the fulfilment of most of the properties that are desirable for fuzzy quantifiers has been described in [11, 13]. Our proposals do not fully fulfil the axiomatic framework presented in these works, but they show a very good behaviour [9], a clear and intuitive semantic interpretation that is based on voting models [2], and it also seems to be possible to modify these proposals for avoiding some counterintuitive behaviours [8].

2 Voting model and quantified sentences evaluation

Voting model [2] understands a fuzzy set $X \in \widetilde{\wp}(E)$ as the result of carrying out a random experiment that is summarized by its membership function μ_X. This interpretation assumes that a set of individuals (voters) take binary decisions about the elements of the referential that fulfil the property represented by X. In this way, membership function $\mu_X(e), e \in E$ indicates the probability that a randomly selected voter v state that e fulfils the property represented by X.

Under this construction, it can also be interpreted that vagueness arises from the different degrees of specificity of voters that are asked to decide which of the elements of the referential set are compatible with linguistic terms. This is similar to assuming that a uniform probability distribution on the specificity levels of the voters exists, or that all specificity levels are equally probable.

Figure 1 shows an example of this interpretation for the values on the referential set *height* that are considered to be *high* by two voters. In this figure, voter v_1 (with specificity level of 0.8) considers elements in $[188, \infty)$ as representatives of the property *high*; voter v_2 (with specificity level of 0.3) considers elements in $[183, \infty)$ to be high. It should be noted that it is natural to relate this interpretation with α-*cuts* (e.g. $high_{0.8} = [188, \infty)$ and $high_{0.3} = [183, \infty)$).

We now go on to make some definitions that make it possible to evaluate fuzzy quantified sentences. Later, we relate these definitions with voting models.

Definition 1 (Semi-fuzzy quantifier). *[11, 13] An s-ary semi-fuzzy quantifier Q on a base set $E \neq \varnothing$ is a mapping $Q : \wp(E)^s \longrightarrow [0,1]$ which assigns a gradual result $Q(X_1, \ldots, X_s) \in [0,1]$ to each choice of crisp $X_1, \ldots, X_s \in \wp(E)$.*

Examples of semi-fuzzy quantifiers are:

$$\text{about\_}\mathbf{5}_E(X_1, X_2) = T_{2,4,6,8}(|X_1 \cap X_2|)$$

$$\text{about80\%\_}ormore_E(X_1, X_2) = \begin{cases} S_{0.5,0.8}\left(\frac{|X_1 \cap X_2|}{|X_1|}\right) & X_1 \neq \varnothing \\ 1 & X_1 = \varnothing \end{cases}$$

where T represents a trapezoidal fuzzy set and S the Zadeh's S-function.

Fuzzy quantifiers are similar to semi-fuzzy quantifiers but taking values in the fuzzy powerset of E (i.e, a mapping $\widetilde{Q} : \widetilde{\wp}(E)^s \mapsto [0,1]$).

Semi-fuzzy quantifiers are much more intuitive and easier to define than fuzzy quantifiers, but they do not resolve the problem of evaluating fuzzy quantified sentences. In order to do so mechanisms are needed that enable us to transform a semi-fuzzy quantifier into a fuzzy quantifier; i.e., a function with domain on the universe of semi-fuzzy quantifiers and range on the universe of fuzzy quantifiers (quantifier fuzzification mechanisms (QFM) mentioned in the introduction):

$$F : (Q : \wp(E)^s \mapsto [0,1]) \mapsto \left(\widetilde{Q} : \widetilde{\wp}(E)^s \mapsto [0,1]\right) \tag{1}$$

Definition 2 (QFM related to P). *[9] Let $Q : \wp(E)^s \rightarrow [0,1]$ be a semi-fuzzy quantifier and $P(\alpha_1, \ldots, \alpha_s)$ a probability density function. We define the quantifier fuzzification mechanism related to P as:*

$$F^P(Q)(X_1,\ldots,X_s) = \int_0^1 \cdots \int_0^1 Q\left((X_1)_{\alpha_1},\ldots,(X_s)_{\alpha_s}\right) P(\alpha_1,\ldots,\alpha_s) d\alpha_1,\ldots,d\alpha_s \tag{2}$$

$P(\alpha_1, \ldots, \alpha_s)$ is a probability density that describes the relationship that exists for every voter among the specificity levels used for defining the terms in the quantified statement. F^P represents the mean of the decisions of the voters.

According to different definitions of the probability density P different mechanisms can be defined. Among others, the following ones can be mentioned:

- **Maximum dependence model.** This scheme arises from assuming that voters are equally specific for all properties:

$$F^{MD}(Q)(X_1, \ldots, X_s) = \int_0^1 Q\left((X_1)_\alpha, \ldots, (X_s)_\alpha\right) d\alpha$$

  This expression, keeping appart differences related to representatives normalisation, generalizes one of the models defined in [6].
- **Independence model.** This scheme [7, 9] arises from assuming that the specificity level of a voter for one property does not constraint his/her specificity level for other properties

$$F^I(Q)(X_1, \ldots, X_s) = \int_0^1 \cdots \int_0^1 Q\left((X_1)_{\alpha_1}, \ldots, (X_s)_{\alpha_s}\right) d\alpha_1 \ldots d\alpha_s$$

Example 1. Let us consider sentence: "*most tall men are blond*", where quantifier $Q =$"most", and fuzzy sets $X_1 =$"tall" and $X_2 =$"blond" take the values:

$$X_1 = \{1, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4\}, X_2 = \{0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3\}$$
$$Q(X_1, X_2) = \begin{cases} \max\left\{4\left(\frac{|X_1 \cap X_2|}{X_2}\right) - 3, 0\right\} & X_1 \neq \emptyset \\ 1 & X_1 = \emptyset \end{cases}$$

The results that are obtained for the two previously presented models are:

$$F^{MD}(Q)(X_1, X_2) = \int_0^1 Q\left((X_1)_\alpha, (X_2)_\alpha\right) d\alpha = 0.3$$
$$F^{I}(Q)(X_1, X_2) = \int_0^1 \int_0^1 D\left((X_1)_{\alpha_1}, (X_2)_{\alpha_2}\right) d\alpha_1 d\alpha_2 = 0.51$$

Previous models cannot be considered as "pure" quantifier fuzzification mechanism in the sense of definition 1, since the integral may not exist for non-finite referentials. Nevertheless, the finite case is sufficient for practical applications.

The methods that are derived from the probabilistic framework here presented do not fulfil all of the axioms that are stated in [11], but they do fulfil most of the important properties that are derived from them [9]. Furthermore, the simple and understandable semantic interpretation underlying the probabilistic framework is an interesting feature that should be pointed out. Some of the properties that are fulfiled by this framework are (due to lack of space, definitions and proofs are omitted): *correct generalization of crisp expressions, external negation, monotonicity, local monotonicity, specificity in quantifiers, induced operators, etc.* Moreover, the maximum dependence model guarantees the fulfilment of *internal meets*, whilst the independence model fulfils *antonymy* and *duality*.

In spite of their general good behaviour none of these probabilistic quantifier fuzzification mechanisms is fully coherent with the interpretation of fuzzy sets we are using. In particular, this happens when the fuzzy sets a quantifier takes as arguments are derived from linguistic terms associated to the same linguistic variable.

In figure 2 examples for explaining the reasons of this lack of coherence for the maximal dependence model are presented. In figure 2a assignation of the elements in the referential to the labels in the term set $\{short, medium, tall\}$ is shown for a voter having an specificity level $\alpha_1 > 0.5$. We can see that some elements are assigned to no term. In figure 2b, assignation for a voter having an specificity level $\alpha_2 < 0.5$ is presented. For this case, some height values are simultaneously assigned to two labels. Both of these examples show situations that do not accommodate the reasonable voting model assumption that all voters should associate a single label to every element in the referential. Similar problems exists for the independence model. The following example shows the problems we have just mentioned when evaluating a fuzzy quantified sentence:

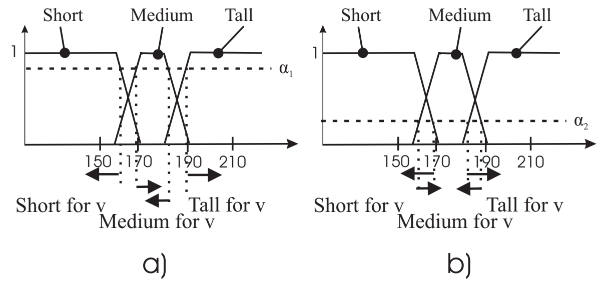

Fig. 2. Incoherence situations for the dependence model.

Example 2. Let us consider the following statement "all individuals are medium or tall", where semi-fuzzy quantifier $all_E : \wp(E) \to [0,1]$ is defined as:

$$all_E(X) = 1 \text{ iff } X = E, X \in \wp(E)$$

Let $E = \{e_1, \ldots, e_{10}\}$, and assume that $height(e_i) = 185, i = 1, \ldots, 10$. Intuitively, this sentence is true for the labels defined in figure 2. But if we calculate the union of *tall* and *medium* with *tconorm* max we obtain

$$F^{MD}(all_E)\left(medium \widetilde{\cup} tall\right) = F^{I}(all_E)\left(medium \widetilde{\cup} tall\right) = 0.5$$

In order to overcome the difficulties just mentioned, in [8] a preliminary definition has been by defining a probabilistic model over formulas involving linguistic terms.

3 Fields of application for fuzzy quantified statements

3.1 Fuzzy Temporal Rules in intelligent control (temporal fuzzy control)

In the field of intelligent control, we have used fuzzy quantified statements within a more general framework known as Fuzzy Temporal Rules (FTRs) [5] in order to implement two behaviours in mobile robotics: wall-following and moving obstacles avoidance [14]. FTRs allow the control action to be taken after considering information on the state of the system (both the robot and its environment) within prior temporal instants and not only the current one, as is usual in fuzzy control. Quantification here plays the role of detecting whether certain situations of interest occur persistently on a given temporal window, in a partially persistent way or on a single temporal point. By means of the analysis of the evolution of state variables throughout temporal references a filtering of noisy sensorial inputs is also achieved, and therefore the control action is more reliable. This is a crucial aspect in robotic applications, where uncertainty about measures (obstacles and/or robot location, speed, ...) due to the ultrasound sensors limitations (crosstalking and other measurement errors) may be decisive.

For the implementation of the wall-following behaviour, a linear velocity controller for the robot was designed. An example of these quantified proposition is: “IF the frontal distance was low *in part of* the last four measurements...”.

In the moving obstacles avoidance behaviour implementation FTRs are useful for taking into account the history of recent values of speed and position of an obstacle and therefore make a good interpretation of what the trend (passing before the robot, letting the robot pass, or indiferent) of this obstacle is. An examples of these quantified sentences is: “collision trend is increasing *throughout* the last second”.

3.2 Modelling of task-oriented vocabularies

Modelling of task-oriented vocabularies [10] allows us to increase expressiveness and aplicability of fuzzy knowledge based systems. Usual examples of use of these vocabularies range from flexible queries to data mining in databases, and also in the previously mentioned field of temporal fuzzy control [5]. By using similar approaches to the ones presented in the field of fuzzy quantification, the modeling of fuzzy specification operators (“*maximum*”, “*minimum*”, “*last*”, ...) and of fuzzy reduction operators (“*mean*”, ...) can be developed.

Let $f : \wp^s(E) \rightarrow \mathbb{R} \cup \{\theta\}$ be an arbitrary function, where θ indicates an undefined situation and the referential set E is supposed finite. On the basis of f we define the function $g^f : \wp^s(E) \times \mathbb{R} \cup \{\theta\} \rightarrow \{0,1\}$ as

$$g^f(X_1,\dots,X_s)(r) = \begin{cases} 0 & f(X_1,\dots,X_s) \neq r \\ 1 & f(X_1,\dots,X_s) = r \end{cases}$$

and then, calculate the probability of r being the evaluation of f over $X_1,\dots,X_s \in \widetilde{\wp}(E)$ for the probability density P as

$$P(r|X_1,\dots,X_s) = \int_0^1 \dots \int_0^1 g^f\left((X_1)_{\alpha_1},\dots,(X_s)_{\alpha_s}\right)(r)\, P(\alpha_1,\dots,\alpha_s)\, d\alpha_1 \dots d\alpha_s$$

Example 3. Suppose that we wish to evaluate the sentence “the last temporal point at which the temperature was high during last minutes”. Let $E = \{e_1,e_2,e_3,e_4,e_5,e_6,e_7\}$ be the temporal referential. Let

$$\begin{aligned} high(E) &= \{0.7/e_1, 0.9/e_2, 0.2/e_3, 0/e_4, 1/e_5, 0.9/e_6, 0.7/e_7\} \\ last\_minutes(E) &= \{0.3/e_1, 0.7/e_2, 1/e_3, 1/e_4, 1/e_5, 0.7/e_6, 0.3/e_7\} \end{aligned}$$

be the sets “high temperatures” and “during last minutes”. In the crisp case we could evaluate the sentence by formulating the function

$$f(X_1,X_2) = \begin{cases} max(X_1 \cap X_2) & X_1 \cap X_2 \neq \varnothing \\ \theta & X_1 \cap X_2 = \varnothing \end{cases}$$

where X_1 represents the “high temperatures” and X_2 “during last minutes”. We define

$$g^f(X_1,X_2)(r) = \begin{cases} 0 & f(X_1,X_2) \neq r \\ 1 & f(X_1,X_2) = r \end{cases}$$

If we assume the independence profile for the extension of f we obtain

$$P\left(e_7 | high, last\_minutes\right) = 0.21, P\left(e_6 | high, last\_minutes\right) = 0.42$$
$$P\left(e_5 | high, last\_minutes\right) = 0.37$$

3.3 Information Retrieval

The application of fuzzy set theory for Information Retrieval (IR) is not mature yet. Although a number of researchers have devoted their efforts to designing flexible retrieval models based on fuzzy sets (see e.g. the very interesting compendia [4]) there exists still a gap, especially at the experimentation level, that makes fuzzy approaches not very popular among the IR community [1]. Most of the works on fuzzy sets for IR did not pay much attention to supply efficient implementations and very few researchers attempted to test their theories against standard benchmarks of the field.

Our approach in this field is at a initial stage. It aims at extending fuzzy query languages and testing such models against standard evaluation datasets (at this moment we are employing a subset of the TREC collection containing more than 175.000 documents). Preliminary results are very promising. Our experiments show clearly that the introduction of quantifiers can lead to improvements in retrieval performance up to 20% of average precision[1] when compared to the vector-space model.

Our fuzzy IR model is based on the quantification models previously described. At the moment, best results were obtained using fuzzy quantifiers for the different statements of the query (title, description, narrative) and, then, aggregating these results by means of either a fuzzy quantifier or a tnorm.

More details on this experimentation including precision versus recall tables are being presented soon. We also aim to introduce more complex queries and more complex quantifiers. In these cases query results are expected to take advantage of the good theoretical behaviour of the probabilistic approach for fuzzy quantification and its capability for modelling different types of quantifiers.

4 Conclusions

In this paper a probabilistic fuzzy quantification model developed in our research group has been presented. Previous results regarding both its definition and applications are described. The main feature of this model is its clear underlying semantics and its flexibility.

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[1] Precision is an standard performance ratio that helps to measure the goodness of a ranking of documents.

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