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A Language for Expressing Fuzzy Temporal Rules

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Abstract

This paper deals with the formal description of what we call Fuzzy Temporal Propositions: propositions with explicitly expressed information of a temporal type. The set of syntactic rules that make a grammar up for defining a language for this kind of propositions is presented. For some of the rules, examples that illustrate the expressive power of this type of knowledge representation are introduced. Semantic criteria and definitions are also introduced through examples in order to show how intuitive results are obtained when a reasoning process is performed on Fuzzy Temporal Rules for those cases.
Keywords: Fuzzy Temporal Propositions, Fuzzy Temporal Knowledge Representation and Reasoning, Formal Language for Fuzzy Temporal Rules.

1 Introduction

Up until the present, applications based on fuzzy rules have constituted the most significant industrial and commercial success of what has been labelled fuzzy technology. Diagnostic, classification and/or pattern recognition systems and, to an even greater degree, fuzzy controllers, are clear examples in which fuzzy rules have had and currently have unquestionable specific importance.

Nevertheless, in spite of the enormous success that the fuzzy set theory has had in these environments, it is also patently clear that the structure of the fuzzy rules that make them up (and as such the **Fuzzy Knowledge Bases** (FKBs) in which they are grouped) does not appear to have evolved in a significant manner since the appearance of the first proposals [19]. FKBs were already described in these proposals as being formed by fuzzy rules that are constructed as the conjuction/disjunction of basic propositions of the type "*X is A*" (e.g., "*Temperature is High*", "*Error is Low*"), and grouped into a very regular and parallel structure, which has clearly differentiated state and control variables.

In some proposals [4, 24] more generic or flexible structures for FKBs have been suggested, by means of chaining between rules, for example. Nevertheless, not too many variations have taken place in the basic structure of the propositions making up the rules. Syntactically, these propositions always bind a linguistic variable with a linguistic value. During the FKB execution process a (numerical or linguistic) value is obtained which estimates the Degree of Fulfillment (DOF) of the propositions. The DOF is habitually calculated by means of simple operations between the membership function that defines the linguistic value and the function which describes the value of the variable which is actually observed in a determined moment. Thus, this value resumes the DOF of a proposition "X *is* A" when it is actually observed that "X *is* $\tilde{A}$".

A large number of applications [10, 12, 18, 20, 21, 23] show portions of knowledge that do not correspond to this very simple scheme of propositions. Thus, for example, in monitoring and/or control environments we may find ourselves with expressions such as "*Temperature has been very low for a few seconds*" or "*Vomiting started later than 15 min after the beginning of the radiation exposure*" [18] or "*Sometime within the last hour Temperature 1 was much greater than Temperature 2*". In all of these examples the time variable is introduced explicitly, playing a central role in the meaning of the proposition, be it as a temporal reference of the events ("*for a few seconds*", "*for the last hour*"), or as a relation between the occurrence of events ("*15 minutes after*"). The structure of these propositions does not fit into the conventional atemporal scheme "X *is* A", at least in an intuitive and direct manner, due to which it becomes necessary to increase the flexibility and possibilities of the fuzzy proposition model. Furthermore, the accurate evaluation of the fulfillment of these propositions demands their suitable modelling from a semantic point of view, giving suitable expressions which allow the calculation of their DOF during their execution process in the FKB.

In [2, 5, 6, 7] some aspects related to the representation and execution of **fuzzy temporal rules** are presented, fuzzy temporal rule (FTR) being understood as one which represents information of a temporal type in an explicit manner. The model there presented permits using temporal references in the propositions either absolute or relative to occurrence of events. All the semantic aspects there described were presented outside the formal and integrational setting proposed in this paper, and, in any case, without reference to the language proposed herein.

Once the model of FTRs has been developed, in this paper we present a formal definition of a grammar that describes **Fuzzy Temporal Propositions** (FTPs), as those making up the FTRs. The grammar contemplates the double perspective of the problem: it provides a syntactic description of the propositions that it models, and a procedure (semantic criteria) for evaluating the DOF of these propositions. The aim is to implement a language, closer to the natural one, which allows experts to describe their knowledge (including the temporal component) in a legible and flexible way. These tasks involve knowledge acquisition and manipulation, by means of a language that "translates" expert's knowledge into a computable model.

2 From Fuzzy Propositions To Fuzzy Temporal Propositions

In [8] we introduced an FTR model endowed with substantial expressive capabilities, and analyzed the FTR execution process. In the following, we introduce the main features of this model of FTRs, and then we present a formal grammar which encompasses the degree of expressiveness of the model.

2.1 Time Ontology

For representing temporal entities, and following [14, 15], we assume a discrete time axis τ, where time point $t_0 \in \tau$ is assumed to be the time origin, and $\forall k \in \mathbb{Z}$ ($\mathbb{Z}$ being the set of integer numbers), $\delta = t_k - t_{k-1}$ is assumed to be a constant.

Basically, we consider that a fuzzy temporal reference or constraint can be described in an absolute manner (e.g. "*at 20.00* "), in a manner relative to the current moment ("*ten minutes ago*") or in a manner relative to the occurrence of an event ("*a little bit after an increase in pressure*", "*between 30 min and 2 hours after the beginning of irradiation*" [18]). From a quantitative and qualitative point of view, it may be an instant or a temporal interval.

Following [14], we understand **fuzzy instant** T as being a possibility distribution μ_T defined over the time axis τ, such that for a time point $t_0 \in \tau$, $\mu_T(t_0)$ represents the possibility of T being precisely t_0. On the other hand, a **fuzzy interval** is defined based on its initial T_S and final T_E instants and its duration D by means of a possibility distribution defined over τ that comprises the time points that are possible after T_S and before T_E.

Furthermore, we consider the basic temporal relationships (qualitative and quantitative) between these entities [1], at the level of instants i and intervals I ($i - i$, $i - I$, $I - I$), and between temporal distances ($D - D$). Some examples of this kind of relationships are: *before, at the end of...*

In any case, we suppose that the temporal entities refer either to the current time point (t_{now}) or to the past. We do not consider in any case expressions that make reference to the future.

2.2 Ontology of facts

We assume that we are operating with discrete signals S, described by means of a function $S(t)$, which represents the history of its crisp values.

Two types of signals are assumed:

- **Observable**: signals whose values are supplied from outside the system, by means of sensors, file readings, etc.
- **Inferible**: signals whose values are supplied by the system itself, by means of inference processes based on prior observations and/or inferences.

We also assume two basic types of facts [11] that can be referred to in the FTRs:

- **Event**: a fact associated to a temporal instant, fuzzy or not (it may be a simple numeric or symbolic value). It is a fact with a null duration (its real duration is inferior to the temporal unit).

- **Episodes:** a fact associated to a temporal interval, fuzzy or not, in which the conditions which identify the fact itself persist. It is a temporal fact with a non null duration.

All the facts (observed or inferred) are associated to absolute times, precise or not. This allows the establishment of a total order between the known facts. On the other hand, it is assumed that the signal histories conform sequentially in time. We are therefore orientating our model towards real time applications.

2.3 Model of Fuzzy Temporal Rules and Propositions

The FTRs in our model take the form:

IF P_1 and P_2 and ... and P_M THEN C_1 and C_2 and ... and C_N
where P_m, $m = 1, \ldots, M$, are propositions of the antecedent part of the rule, and C_n, $n = 1, \ldots, N$, of the consequent part of the rule (conclusions), which take the form $C_n : (S_n, V_n, T_n)$, S_n being an inferible signal, V_n the value (represented by a non-temporal fuzzy set) inferred for this signal and T_n the time (fuzzy instant or interval) associated to the inferred value.

For the antecedent part, we contemplate a proposition format with a great degree of expressiveness, which enables us to represent facts in which information of a **spatial** (in the sense of non-temporal) or a **temporal** type that is linked to them may be fuzzy, and be given in a manner that is absolute or relative to other facts. Furthermore, we allow the introduction of operators associated, for example, to quantification, fact specification or reduction processes, as is commented on below.

In the most general case, a proposition may contain different kinds of **constraints** acting on a signal:

- **Value Constraints:** spatial value constraints on the signal, which may be given in an absolute manner (e.g. "*high*"), or related to another spatial reference value (e.g. "*greater than temperature in heater 2*").

- **Temporal Constraints:** absolute or relative temporal constraints, examples of which are "*throughout the last half an hour*", "*a little after 3 o'clock*", "*before the maximum value of pressure*". This requires the consideration of the different relations between temporal entities (instants, intervals) [1].

  Temporal constraints may also set a temporal context of signal evaluation, establishing a temporal window (interval) within which the proposition will be evaluated.

- **Operators:** belonging to one of the following types:

  - **Quantification operators**: In some cases it may be of interest that fuzzy temporal propositions are quantified, as in "*The temperature has been high during the majority of the last half an hour*".The use of quantifiers permits, for instance, to model the persistence of a value throughout a temporal interval (indicating whether the complete fulfillment in that interval is required, or some partial fulfillment is enough). These quantifiers can operate either on the spatial or temporal parts of the proposition (*all, the majority, between 3 and 5, approximately half,...*).

- **Specification operators:** in order to select one candidate from amongst various, according to a specific criterion: spatial (*maximum, minimum,...*) and/or temporal (*first, last,...*).
- **Reduction operators:** In this case the constraint operates on the spatial values that are observed or inferred for the proposition, in order to return a new value, calculated on the basis of the former ones (*mean value, accumulated value,...*).

The proposed model, in the simplest of cases, would allow propositions with the structure, already mentioned in prior examples, "*X is A in T*", where T is the time reference and "*X is A*" is the atemporal (or, equivalently, "spatial") part of the proposition. Obviously, more complicated cases than this one are contemplated. The propositions may be made up by relations between variables ("*Temperature 1 is much higher than Temperature 2*") with different degrees of structural complexity ("*Temperature 1 in the last few seconds was much higher than Temperature 2 throughout the last few minutes*"). More generic structures, in which the temporal reference of the proposition and its spatial part are rewritten in a much more elaborate manner, can be described: "*Pressure 1 was much higher than Pressure 2, a little bit after that Temperature 1 was much lower than Temperature 2*", "*Temperature is low while Pressure is high*". Related to the use of different operators, we also may have expressions like *"The majority of the temperature values throughout the last few seconds have been high"* (quantification), or *"The mean value for the temperature over the last 48 hours was moderate"* (reduction).

In order to formalize the description of all these cases, and potential combinations of others, a grammar is presented in the next section. The description of the grammar comprises all rules that permit the construction of a temporal specification language.

3 A Grammar For Fuzzy Temporal Propositions

Once we have defined the fundamentals of our FTR model, the next step is to define a language which allows the adequate projection of the linguistic specification of the rules made by an expert. This language should collect all the semantic expressiveness of the expert's knowledge.

3.1 Rewriting rules of the grammar

We introduce a grammar describing the structure of FTPs, which contemplates all the elements mentioned above. The complete set of syntactic rules, combining temporal and spatial values and constraints, is shown in Table 1, using the BNF (Backus-Naur Form) metalanguage: a **non-terminal** symbol of the language is defined from a sequence of **terminal** and/or non-terminal symbols. In the definition of these rules, the following metasymbols are used: '::=' is the rewriting metasymbol, '[]' indicate the optionality of their content, '|' separates the mutually exclusive options delimited by '()', and '<>' indicate that their content is not a terminal element.

Table 1: Rewriting rules of the grammar (using BNF metalanguage).

<table>
<tr><td>(R1)</td><td><Propositional Clause>::= (<Proposition> |
| <Proposition><Spatial Relation><Proposition>) [<Temporal Constraint>]</td></tr>
<tr><td>(R2)</td><td><Proposition>::= <Generalised Signal> <Temporal Instant> |
| (<Operator> | <Quantifier>) <Gen. Signal> [<Val. Constr.>] <Temporal Interval></td></tr>
<tr><td>(R3)</td><td><Generalised Signal>::= <Signal> [<Value Constraint>]</td></tr>
<tr><td>(R4)</td><td><Signal>::= TEMPERATURE | PRESSURE | ...</td></tr>
<tr><td>(R5)</td><td><Value Constraint>::= HIGH | LOW | TALL | MORE THAN 30 | ...</td></tr>
<tr><td>(R6)</td><td><Spatial Relation>::= GREATER THAN | LOWER THAN | SIMILAR TO |...</td></tr>
<tr><td>(R7)</td><td><Temporal Constraint>::= <Temporal Instant> | <Temporal Interval></td></tr>
<tr><td>(R8)</td><td><Temporal Instant>::= [<Instant-Instant Relation>]<Instant>|
| <Instant-Interval Relation><Interval></td></tr>
<tr><td>(R9)</td><td><Temporal Interval>::= [<Interval-Interval Relation>] <Interval> |
|<Interval-Instant Relation> <Instant></td></tr>
<tr><td>(R10)</td><td><Instant-Instant Relation>::= [<Instant-Instant Relation>] (<Time Distance> |
| [APPROXIMATELY] EQUAL)</td></tr>
<tr><td>(R11)</td><td><Instant-Interval Relation>::= [<Instant-Interval Relation>] (<Time Distance> |
[<Instant-Instant Relation>] (BEGINNING | END) | BELONGS)</td></tr>
<tr><td>(R12)</td><td><Time Distance>::= [<Time Extent>] (AFTER | BEFORE)</td></tr>
<tr><td>(R13)</td><td><Time Extent>::= <Temporal Quantity> [<Temporal Unit>]</td></tr>
<tr><td>(R14)</td><td><Temporal Quantity>::= $k \in \mathbb{Z}$| LITTLE | MUCH | AT LEAST SOME |...</td></tr>
<tr><td>(R15)</td><td><Temporal Unit>::= ...| SEC | MIN | HOUR |...</td></tr>
<tr><td>(R16)</td><td><Instant>::= <Direct Instant> | <Propositional Clause></td></tr>
<tr><td>(R17)</td><td><Direct Instant>::= $t \in \tau$| NOW | TODAY | AT NOON |...</td></tr>
<tr><td>(R18)</td><td><Interval-Instant Relation>::= [<Interval-Instant Relation>] (<Time Distance> |
[APPROXIMATELY] (UNTIL | FOLLOWS) | INCLUDES)</td></tr>
<tr><td>(R19)</td><td><Interval-Interval Relation>::= [<Interval-Interval Relation>] (<Time Distance> |
([APPROXIMATELY] (STARTS WITH | STARTED BY | AT | FINISHES WITH |
| FINISHED BY | UNTIL | FOLLOWS)) | | INCLUDES | DURING |
|OVERLAPS (SINCE) | OVERLAPPED BY (UP TO))</td></tr>
<tr><td>(R20)</td><td><Interval>::= <Direct Interval> | <Propositional Clause></td></tr>
<tr><td>(R21)</td><td><Direct Interval>::= (<Instant>, <Instant>, <Time Extent>) |
|BETWEEN<Instant>AND<Instant> | YESTERDAY | TODAY |...</td></tr>
<tr><td>(R22)</td><td><Operator>::= <Reduction Operator> | <Specification Operator></td></tr>
<tr><td>(R23)</td><td><Reduction Operator>::= MEAN_VALUE | ACCUMULATED_VALUE | ...</td></tr>
<tr><td>(R24)</td><td><Specification Operator>::= MAXIMUM | MINIMUM | LAST | ...</td></tr>
<tr><td>(R25)</td><td><Quantifier>::= [APPROXIMATELY] (ALL | THE_MAJORITY |
| BETWEEN_3_AND_5 | A_HALF | ...)</td></tr>
</table>

It can be seen in the table how the first three rules establish the general structure of a propositional clause, including all the elements we define in the previous section (when describing the model of FTRs): temporal and value constraints, as well as, possibly, some kind of operators on these elements. In rules R5-R6, the structure of the value constraints is described, allowing for absolute and/or relative constraints on the signals. Following the model in [1], rules R7 to R21 deal with temporal constraints and temporal relations between temporal entitites (instants, intervals, time extents or distances), enabling a great flexibility in the use of temporal references in the propositions. Finally, rules R22 to R25 correspond to the description of the different kinds of operators: reduction, spatial and temporal specification, and quantification.

With the aim of illustrating the expressive capabilities of the proposed language, we will present a set of examples where some propositions in natural language are translated according to this grammar. In order to point out the degree of expressiveness of the grammar, some of the examples show how the semantics associated to the syntactic expressions can be modelled, according to the considerations stated in this section. Although this corresponds to a semantic modelling of the language, we include this part for the sake of clarity of the examples and to show how intuitive and correct results can be obtained.

Example 1: "*Temperature was high throughout/in the last 30 minutes*".

A central concept in temporal reasoning is the one of **persistence** [6], which is related to the use of fuzzy intervals as temporal references in propositions. This concept responds to the intuitive idea that a proposition in which it is said, for example, "*Temperature was high throughout the last 30 minutes*", does not have an identical meaning to a proposition that indicates "*Temperature was high in (at any moment within) the last 30 minutes*". In the former, the occurrence of the non-temporal part "*Temperature was high*" is required for <u>all</u> the time points in the time interval, while in the latter, the proposition is true if this occurs for <u>any</u> time point in the interval. This situation is characterised in our grammar by quantification operators acting on the support of the temporal constraint in the proposition, giving rise to different semantic rules which realise the accurate evaluation of the meaning in each case (depending on the specific operator). The syntactic analysis of these propositions is represented in Fig. 1.

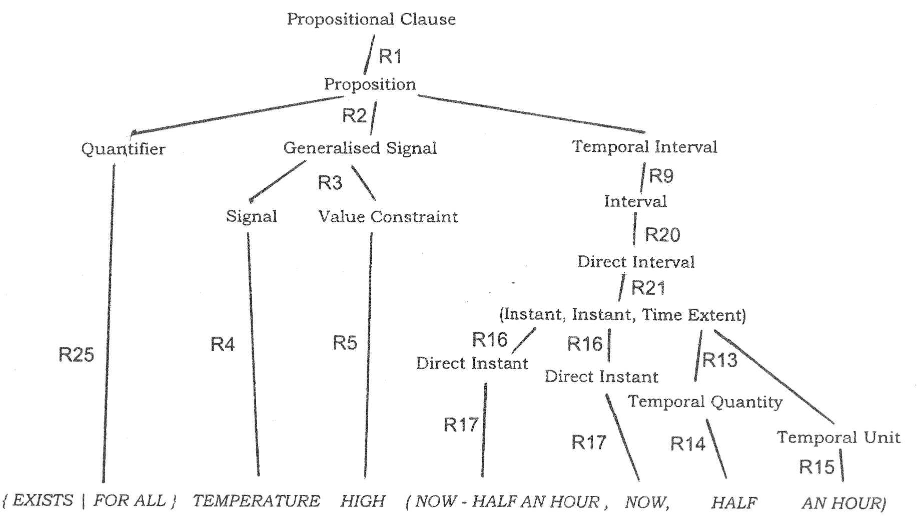

Figure 1: Syntactic analysis of propositions in Example 1.

By way of illustration, and in a totally qualitative manner, Figure 2 shows the differences in meaning between the two extreme situations (quantifiers EXISTS and FOR_ALL, rule R25), for this simple example.

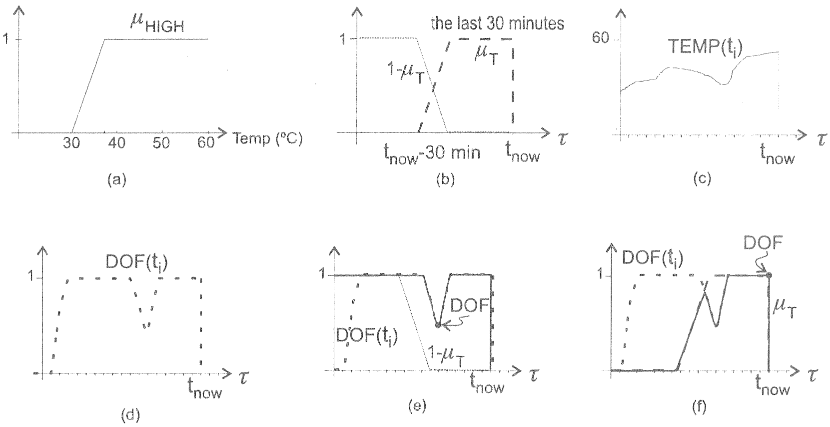

Figure 2: Calculation of DOF for the two propositions in Example 1.

We assume that membership function μ_{HIGH} in Fig. 2(a) defines the linguistic value HIGH for the linguistic variable TEMPERATURE, time membership function μ_T in Fig. 2(b), the time interval THE LAST 30 MINUTES, and the recent history of Temperature observed values $TEMP(t_i)$ is the one described in Fig. 2(c). In order to calculate the DOF, in the first place, a **linguistic filtering** process [22] (Fig. 2(d)) produces the history of the DOF of the non-temporal part of the proposition ("*Temperature was high*"):

$$DOF(t_i) = \mu_{HIGH}(TEMP(t_i)), \ \forall t_i \in \tau \tag{1}$$

Simple maximum and minimum operations [6] between this DOF history and time interval μ_T provide the DOF of the proposition. For the persistence situation (universal quantification: "*Temperature was high throughout the last 30 minutes*") we have (Fig. 2(e)):

$$DOF = \bigwedge_{t_i \in \tau} DOF(t_i) \vee (1 - \mu_T(t_i)) = 0.4 \tag{2}$$

and for the non-persistence situation (existential quantification: "*Temperature was high in the last 30 minutes*") (Fig. 2(f)):

$$DOF = \bigvee_{t_i \in \tau} DOF(t_i) \wedge \mu_T(t_i) = 1 \tag{3}$$

These definitions of FOR_ALL ($\forall$) and EXISTS ($\exists$) quantifiers are adapted from any of the usual models of non-temporal fuzzy quantifiers [13]. For both of these cases the weight of time points is proportional to their membership degree to μ_T. It can be seen how, in the persistence case (Fig. 2(e)) the calculated DOF is always lower than or equal to the DOF for the non-persistence case (Fig. 2(f)), as could be expected, since the former is a more restrictive condition. In order to include a correct representation of all possible quantifiers, a quantification model has to be described. This remains an open issue, since all of the fuzzy approaches in this field have proven to partially fail [3, 17].

Regarding other operators, like reduction or specification ones, some considerations must be taken into account when describing their behaviour. Such is the case, for instance, of temporal specification operator "last" (R24), whose interpretation should be different of that usual in databases (the last entry in a queue). The meaning in this model is not the crisp one, since it must also incorporate both the temporal and spatial aspects in its definition (we should be able to model expressions like "*the last value of high temperature values*", where not only the temporal, but also the value constraint must be considered). This is a behaviour that, to the best of our knowledge, has not been dealt with in the literature.

Independently of the actual semantic expressions for the operators, more complex propositions involving (temporal or value) constraints on the result of applying these operators, can be solved by means of simple operations of conjunction and fuzzy sums. An example of this situation is "*Pressure was less than 760 mm a little before the last value of high temperature*".

Example 2: "*Temperature was high at some instant approximately between 15 min. and 1 h. after the beginning of the irradiation*"

The fact that a temporal reference for a proposition can be given in a relative manner is another important representative capability of the model proposed. This is shown in a quantitative manner by means of this example (adapted from [18]).

The semantics associated with the rewriting rules permits the calculation of the DOF for the proposition, following the steps illustrated for the previous example in Fig. 2. Nevertheless, in this new example, as we mentioned, the time interval is not explicitly given, rather it makes reference to an event (BEGINNING_OF_THE_IRRADIATION), represented by its instant of occurrence. In order to obtain the explicit representation for the temporal reference (μ_T), the "Time distance" (rule R12) BETWEEN APPROXIMATELY 15 AND 60 MINUTES AFTER is calculated from the combination of AT LEAST APPROXIMATELY 15 MINUTES AFTER and UNTIL APPROXIMATELY 1 HOUR AFTER. This process is shown in Fig. 3, where t_0 is the time origin (time for the first observed value of TEMPERATURE), t_I the instant when the BEGINNING_OF_THE_IRRADIATION occurs, and t_{now} the current instant. A discretization step $\delta = 5min$ is assumed.

We assume the history of temperature values is the one shown in Fig. 2(c) for the previous example:

Temp(t_i)={27,30,35,38,38,40,45,43,42,41,39,35,33,41,46,50,51,52,53},

$t_i \in [t_0, t_{now}]$

As only the time points belonging to the support of μ_T need to be taken into account, this set is reduced to:

Temp(t_i)={38,40,45,43,42,41,39,35,33,41,46,50}

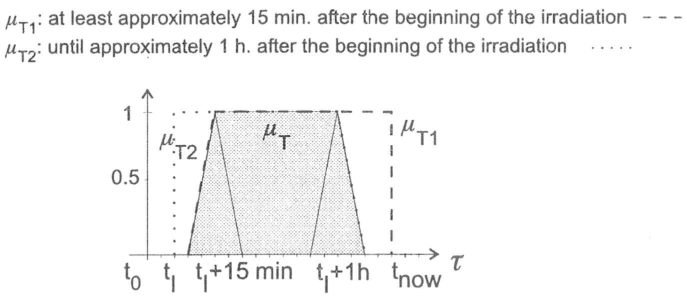

Figure 3: Calculation of the temporal reference $\mu_T = \mu_{T1} \wedge \mu_{T2}$ for proposition in Example 2.

These values must be linguistically filtered with value HIGH, in order to obtain:

DOF(t_i)={1,1,1,1,1,1,1,0.7,0.4,1,1,1}

with the following temporal reference (Fig. 3):

$\mu_T(t_i)$={0.5,1,1,1,1,1,1,1,1,1,1,0.5}

Since the proposition is assumed to have a meaning of non-persistence (this is the default case, when no quantification operator is present), the DOF is calculated according to Eq. (3):

$\text{DOF}=\bigvee(0.5,1,1,1,1,1,1,0.7,0.4,1,1,0.5) = 1$

Example 3: "*Temperature in heater 1 has been very much greater than temperature in heater 2 during the last few minutes*".

Figure 4 shows how this complex proposition, involving two interrelated signals, is represented by means of the rewriting rules. In this case, the spatial values of the two signals are related through a spatial comparator.

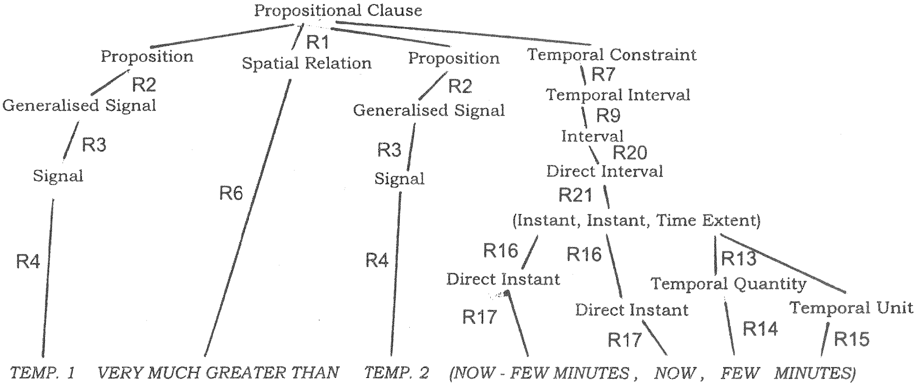

Figure 4: Syntactic analysis of proposition in Example 3.

Example 4: *"Pressure was high a little before temperature was low at some point during the last half an hour".*

In this last example, the temporal reference is given through a temporal constraint on the relation between the spatial values of two signals, PRESSURE and TEMPERATURE.

Figure 5 shows how this proposition is modelled in the grammar according to the instantiation of the corresponding rewriting rules.

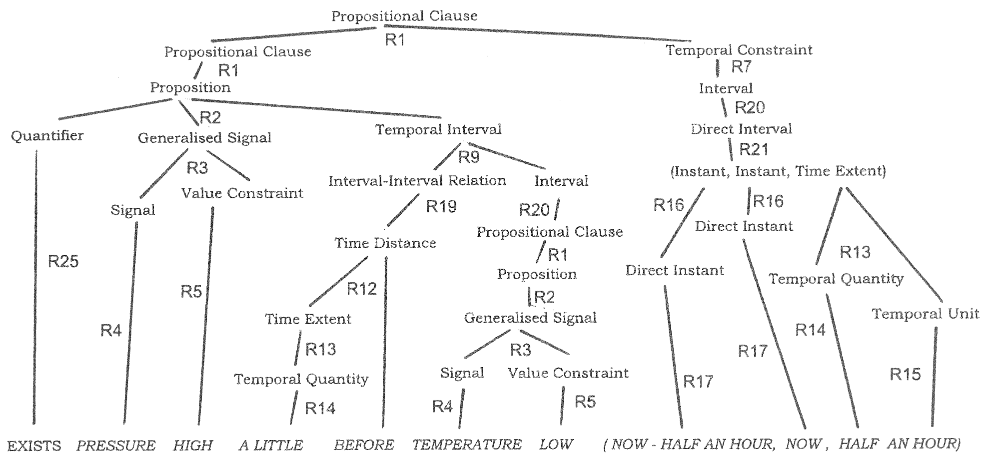

Figure 5: Syntactic analysis of proposition in Example 4.

Figure 6(a) shows the history of recent TEMPERATURE values $S(t)$, assuming $\delta = 5min$. The temporal relationship A_LITTLE_BEFORE is assumed to be defined as the possibility distribution shown in Fig. 6(b).

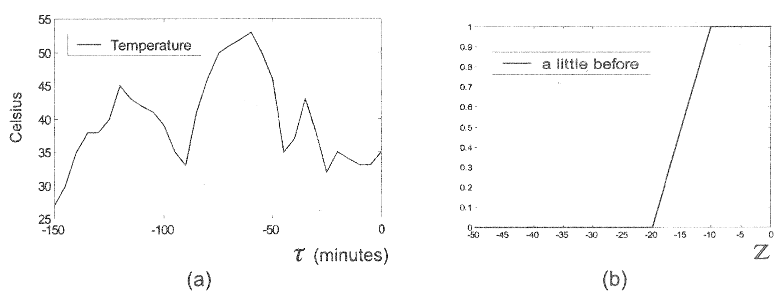

Figure 6: (a) History of TEMPERATURE values for proposition in Example 4. In the time axis, we are assuming t_{now} as the time origin ($t = 0$), in order to make the representation more simple. (b) Definition of temporal relationship A LITTLE BEFORE.

Temperature values are matched with the value constraint LOW in order to obtain the partial DOF shown as *Temp. low* in Fig. 7(a), where the representation of the temporal constraint THE_LAST_HALF_AN_HOUR is also given. IT represents the result of considering both the membership to the temporal constraint THE_LAST_HALF_AN_HOUR and the value constraint LOW for the proposition describing the signal TEMPERATURE. Furthermore, IT provides the temporal reference for the evaluation of the proposition associated to the signal PRESSURE.

The process of calculating the fulfillment of the temporal constraint T for proposition PRESSURE WAS HIGH is shown graphically in Fig. 7(b). The temporal reference for this proposition is obtained as the fuzzy addition between this possibility distribution and the temporal reference IT.

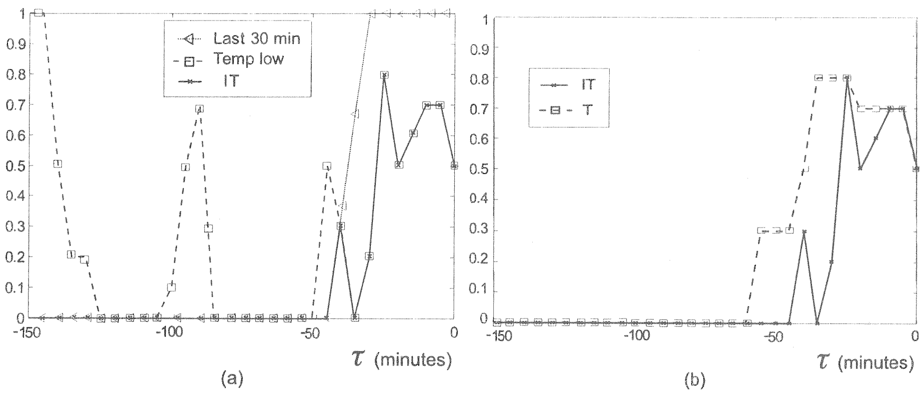

Figure 7: (a) Calculation of temporal reference IT in Example 4, combining TEMPERATURE WAS LOW and THE_LAST_HALF_AN_HOUR. (b) Calculation of the temporal reference T for proposition PRESSURE WAS HIGH: A_LITTLE_BEFORE IT.

4 Discussion

The generalization of fuzzy rules to FTRs, which allows an explicit representation and handling of time, can, no doubt, contribute to extending the application domain of fuzzy logic, or at least, to making the design of solutions for problems associated to dynamic systems and processes easier.

In this work we introduce a grammar associated to a model of FTRs that attempts to represent the semantic expressiveness of expert knowledge. The expressive capacity of the model and how it directly deals with imprecision and uncertainty linked to information and knowledge, make it, in our opinion, very promising for application in environments such as process control or monitoring. We have tried to illustrate the descriptive potential of Fuzzy Temporal Propositions under this grammar by means of a number of examples. The formal description of the grammar in which they are included has been introduced, including some aspects of the semantic part, which allows the calculation of the DOF of the propositions that are syntactically correct. The design of the grammar contemplates a large number of cases of interest, at the same time as completely encompassing totally atemporal propositions.

We are at present studying the different proposals for the semantic representation of all of the rules here described [9]. An important aspect of this task is the representation of quantification operators, which provide the possibility of graduating spatial or temporal persistence [6], which, amongst other things, allows the modelling of propositions such as "X is A in part of T". Reduction and specification operators are the other classes of operators we are aiming to semantically model. This is a very important task that has to be faced, and for which some proposals (for non-temporal applications) have been described in the literature. Most of them, as it happens for the quantification operators, have been shown to fail as adequate models [3, 17].

Another aspect of interest that we are approaching is the incorporation into the model of what we call Fuzzy Temporal Profiles [16], which permit the evaluation of tendencies in variables over time. The inclusion of these and the complete formalisation of the grammar that we outline here will enable us to clearly advance in increasing the expressive capacity of fuzzy propositions, which will permit the opening up of new application environments for fuzzy rule-based systems.

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