# Fuzzy Temporal Rules: A Rule-based Approach for Fuzzy Temporal Knowledge Representation and Reasoning

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**Abstract.** In this paper a model for the representation and execution of a type of fuzzy control rules that explicitly handle time variable (Fuzzy Temporal Rules) is presented. The model here described permits using temporal references for the occurrence of events. These references are allowed to be described either in an absolute manner or related to occurrence of other events. A complete grammar that formally describes the representation language of the model is presented, together with its semantic counterpart. This knowledge and reasoning representation model is illustrated by means of a number of examples that show the substantial increase in the expresiveness that FTRs exhibit when compared to usual fuzzy control rules.

**Keywords:** fuzzy temporal rules, temporal reasoning, knowledge representation and reasoning

## 1 Introduction

The control environment, in which the fuzzy set theory has been applied with undoubted success, is essentially an application field in which variables are dynamic and changeable. In spite of this, it is curious to note that there has been a tendency, it would seem, to opt for simple and basic fuzzy reasoning models, without paying hardly any attention to the possibility of representations of knowledge that explicitly manipulate time and reason on it. In our opinion, this noticeably limits the possibilities for the expansion of fuzzy reasoning to applications which demand more expressive forms of dealing with expert knowledge. The greater complexity required at an operational level by fuzzy temporal rules (FTRs) - fuzzy temporal rule being understood as one which represents information of a temporal type in an explicit manner - as opposed to simple fuzzy rules may be, without doubt, one of the causes of this. On the other hand, the fact that there are still no well-defined models nor computer aid tools for the design of FTR-based systems, means that the use of this form of knowledge representation is very limited, which, in turn, limits the interest of the research and technical communities in FTRs.

In this work, our objective is to present a formal FTR model, which may be used for representation of and reasoning on fuzzy temporal knowledge, and which is endowed with substantial expressive capabilities. We introduce the formal definition of a grammar that describes Fuzzy Temporal Propositions (FTPs), as those making up the FTRs. The aim is to implement a language, close to the natural one, which allows experts to describe their knowledge (including the temporal component) in a legible and flexible way. We then analyze the FTR execution process, describing the semantic expressions associated to the syntactic rules in the proposed grammar, including some examples by way of illustration.

Some of the aspects related to the model of FTPs, mainly concerning the semantics associated with part of the propositions here formalised, can be found in [2,4,5], although outside the formal and integrational setting proposed in this paper.

### 1.1 Some Approaches to the Concept of Fuzzy Temporal Rule

The first attempt to formalize the representation of fuzzy time by means of intervals associated to fuzzy sets was described in [17]. However, arguably the first formal work with regard to the formalization of the concept of fuzzy time is that of [9], where the foundations for the representation of imprecision and uncertainty in temporal knowledge with the theory of possibility were laid. With some variants, the concepts there introduced and the manner of representing them appear in the majority of works relating to fuzzy time.

With regard to models of fuzzy temporal reasoning (fuzzy reasoning that explicitly takes into account the presence of temporal references), these fundamentally deal with constraint network models [3,12] and fuzzy rules which, in one way or another, explicitly introduce time as another decision variable. In [14], a model for fuzzy proposition and temporal reasoning, that deals with three forms of temporal relationship is presented. The model is not capable of handling persistence over time of conditions of the value of the variables that are handled, although all basic temporal relations [1] amongst temporal entities (time instants or intervals) are dealt with.

In [16], the concept of "time dependent fuzzy set" is introduced, whilst in [11] a reasoning method that incorporates a vague time delay into fuzzy *if-then* rules is proposed. Except for the specific mathematical support with which rules are evaluated, this is a similar proposal to [7], which deals with establishing a validity time for the result inferred by a rule, an idea which is also present in the faded temporal fuzzy controller described in [10]. [15] puts forward a model for the representation and manipulation of vague time, based on the theory of possibility and following the temporal entity representational scheme of [9].

## 2 Model of Fuzzy Temporal Propositions

In the following, we present the main features of our model of FTRs, and also a formal grammar which encompasses the degree of expressiveness of the model. A first approach to this problem was presented in [6].

### 2.1 Time Entities

For representing temporal entities, and following [9], we assume a discrete time axis $\tau$, that is discretized in time points $t_n$, $n \in \mathbb{N}$. Time point $t_0 \in \tau$ represents the time origin, and $\forall n \in \mathbb{N}\ \delta = t_n - t_{n-1}$ is assumed to be a constant. The following temporal entities [3] are also considered:
- **Instant** $i$. Represented as a normalized and unimodal possibility distribution $\mu_i(t)$ that represents the possibility of $i$ being precisely time point $t \in \tau$.
- **Temporal extent (duration)** $D$. It represents quantities of time. A fuzzy temporal extent or duration is represented by a possibility distribution $\mu_D$ over the set of integer numbers $\mathbb{Z}$ (assuming they represent units of time $\delta$).
- **Interval** $I(i_b, i_e, D)$. It is described by means of a possibility distribution $\mu_I$ that is calculated as in [9], according to the possibility theory. For example, if the beginning $i_b$ and ending $i_e$ instants are known, $\mu_I$ comprises the time points that are possibly after $i_b$ and before $i_e$. In a similar way it is calculated when either $i_b$ or $i_e$ and duration $D$ are known.

Furthermore, we consider the basic temporal relationships (qualitative and quantitative) [3] between these entities, at the level of instants and intervals ($i - i$, $i - I$, $I - I$), and between temporal extents ($D - D$). We assume that all these relationships between temporal entities can be reduced to relationships between time points and temporal extents. Some examples of this kind of relationships are: *before, at the end of...* We do not consider in any case expressions that make reference to the future.

In this manner, we may have absolute and relative temporal entities. We should strictly consider that the specification of time associated to a fact in absolute terms is any one relative to a fixed entity of time, whilst a specification of time associated to a fact in relative terms will be the one that depends on a temporal entity that is linked to the occurrence of other facts.

For instance, we may have an instant $i$ which is given by a reference to a fact occurring at time $i_{ref}$, bearing in mind that it always has to be prior (or equal) to $t_{now}$, the current time point. In this case, we have: $i = D \oplus i_{ref}$, and using fuzzy addition operator $\oplus$, this is defined as:

$$\forall t \in \tau, \mu_{D \oplus i_{ref}}(t) = Sup_{t=t'+k\delta} min\left(\mu_D(k), \mu_{i_{ref}}(t')\right), t' \in \tau, k \in \mathbb{Z} \qquad (1)$$

### 2.2 Model of Fuzzy Temporal Rules and Propositions

We assume that we are operating with discrete signals $S$, described by means of a function $S(t)$, which represents the history of crisp values associated to a variable, together with the occurrence time points of each one of the aforementioned values. This is what most real-time applications [13] demand.

Two types of signals are defined: **observed** signals, whose values are supplied from outside the system, by means of sensors, file readings, ... and **inferred** signals, whose values are supplied by the system itself, by means of inference processes based on prior observations and/or inferences.

FTRs take the form:

IF $PC_1$ and $PC_2$ and ... and $PC_M$ THEN $C_1$ and $C_2$ and ... and $C_N$

where $PC_m$, $m = 1,...,M$, are propositions of the antecedent part of the rule, and $C_n$, $n = 1,...,N$, of the consequent part of the rule (conclusions), which take the form $C_n$: $S_n$ is $V_n$ in $T_n$, $S_n$ being an inferible signal, $V_n$ the value (represented by a non-temporal fuzzy set) inferred for this signal and $T_n$ the time (fuzzy instant or interval) associated to the inferred value. For the antecedent part, a **fuzzy temporal proposition** consists of a signal and a set of different constraints on it:

- **Value Constraints** ($VC$). Spatial (non temporal) value constraints on the signal, which may be given in an absolute manner ($VC(u) = \mu_{VC}(u)$), or related to another spatial reference value $V_{ref}$ ($VC(u) = (\Delta \oplus \mu_{V_{ref}})(u)$). For example, as in "*high*" or "*much greater than pressure in heater 2*"

- **Temporal Constraints** ($TC$). Absolute or relative temporal constraints over the set of temporal points where the signal will be evaluated. They may be instants ($TCi$) or intervals ($TCI$). Examples of this type of constraints are "*throughout the last half an hour*", "*before the maximum value of pressure*".

- **Temporal Context**. In some cases a temporal constraint may act as a temporal context for signal evaluation, establishing a temporal window (interval) within which the proposition will be evaluated, as in "*a few minutes ago*" or "*the last half an hour*"

We may also have **operators** belonging to one of the following types:

- **Quantifiers** ($O_Q$): *all, the majority, between 3 and 5, approximately half,...* In some cases it may be of interest to quantify the fuzzy temporal propositions, as in "*the temperature has been high during the majority of the last half an hour*".

- **Specification operators** ($O_S$): *first, last, maximum, minimum,...* These select one candidate from amongst various, according to a specific criterion (spatial and/or temporal).

- **Reduction operators** ($O_R$): *mean value, accumulated value,...* In this case the constraint operates on the spatial values that are observed or inferred for the proposition, in order to return a new one, calculated on the basis of the former ones.

In order to formalize the description of which combinations of these elements are allowed, a grammar is presented in the next section. The description of the grammar comprises all rules that permit the construction of a temporal specification language.

# 3 A Grammar for Fuzzy Temporal Rules

## 3.1 Rewriting Rules of the Grammar

We introduce a grammar describing the structure of FTPs, which contemplates all the elements mentioned above. The complete set of syntactic rules, combining temporal and spatial values and constraints, is shown in Table 1, using the BNF (Backus-Naur Form) metalanguage.

It can be seen in the table how the first three rules establish the general structure of a propositional clause $PC$. Rules R5-R6 deal with the different types of value constraints which may act on the signal: absolute and/or relative constraints. In rules R7 to R21 the structure of the temporal constraints and temporal relations between temporal entitites (instants, intervals, time extents) is described, enabling a great flexibility in the use of temporal references in the propositions. Finally, rules R22 to R25 correspond to the description of the different kinds of operators: reduction, spatial or temporal specification, quantification.

## 3.2 Semantic Expressions Associated to the Syntactic Rules

The next step after defining the rewriting rules for a grammar for FTPs is to describe their semantic counterpart: how the propositions are evaluated during the process of execution of the FTRs, in order to obtain a degree of fulfillment(DOF) for a FTP. In principle, for each one of the constraints present in a proposition an associated DOF can be obtained. This DOF indicates the degree in which this constraint is verified according to the *evaluation instance*, understood as the set of data considered (spatial and temporal values and, where appropriate, associated DOFs). The evaluation time for a rule will be the current time point $t_{now}$, which is used as the reference time point for the consequent part of the rule. Furthermore, it should be borne in mind that during the evaluation of propositions in real time, the evaluation scenario will alter as time advances. In the case of there being a temporal context defined explicitly for the proposition, different evaluation instances may be obtained from it, so that the remaining constraints will be evaluated on each one of them. In the absence of other explicit criteria, it will be considered that the result of the evaluation of a proposition in a given temporal context is the one that is obtained on the basis of the best of the possible evaluation instances.

Table 2 shows the semantic rules corresponding to the grammar in Table 1.

**Table 1.** Rewriting rules of the grammar (using BNF metalanguage).

<table>
<tr><td>(R1)</td><td><Propositional Clause>::= (<Proposition> |<br>| <Proposition><Spatial Relation><Proposition>)[<Temporal Constraint>]</td></tr>
<tr><td>(R2)</td><td><Proposition>::= <Generalised Signal> <Temporal Instant> |<br>| (<Operator> | <Quantifier>) <Generalised Signal> [<Value Constraint>]<br><Temporal Interval></td></tr>
<tr><td>(R3)</td><td><Generalised Signal>::= <Signal> [<Value Constraint>]</td></tr>
<tr><td>(R4)</td><td><Signal>::= TEMPERATURE | PRESSURE | ...</td></tr>
<tr><td>(R5)</td><td><Value Constraint>::= HIGH | LOW | TALL | MORE THAN 30 | ...</td></tr>
<tr><td>(R6)</td><td><Spatial Relation>::= GREATER THAN | LOWER THAN | SIMILAR TO |...</td></tr>
<tr><td>(R7)</td><td><Temporal Constraint>::= <Temporal Instant> | <Temporal Interval></td></tr>
<tr><td>(R8)</td><td><Temporal Instant>::= [<Instant-Instant Relation>] <Instant> |<br>| <Instant-Interval Relation> <Interval></td></tr>
<tr><td>(R9)</td><td><Temporal Interval>::= [<Interval-Interval Relation>] <Interval> |<br>| <Interval-Instant Relation> <Instant></td></tr>
<tr><td>(R10)</td><td><Instant-Instant Relation>::= [<Instant-Instant Relation>]<br>(<Time Distance> | [APPROXIMATELY] EQUAL)</td></tr>
<tr><td>(R11)</td><td><Instant-Interval Relation>::= [<Instant-Interval Relation>]<br>(<Time Distance> | [<Instant-Instant Relation>]<br>(BEGINNING | END) | BELONGS)</td></tr>
<tr><td>(R12)</td><td><Time Distance>::= [<Time Extent>] (AFTER | BEFORE)</td></tr>
<tr><td>(R13)</td><td><Time Extent>::= <Temporal Quantity> [<Temporal Unit>]</td></tr>
<tr><td>(R14)</td><td><Temporal Quantity>::= $k \in \mathbb{Z}$| LITTLE | MUCH | AT LEAST SOME |...</td></tr>
<tr><td>(R15)</td><td><Temporal Unit>::= ...| SEC | MIN | HOUR |...</td></tr>
<tr><td>(R16)</td><td><Instant>::= <Direct Instant> | <Propositional Clause></td></tr>
<tr><td>(R17)</td><td><Direct Instant>::= $t \in \tau$| NOW | TODAY | AT NOON |...</td></tr>
<tr><td>(R18)</td><td><Interval-Instant Relation>::= [<Interval-Instant Relation>]<br>(<Time Distance> | [APPROXIMATELY] (UNTIL | FOLLOWS) |<br>| INCLUDES)</td></tr>
<tr><td>(R19)</td><td><Interval-Interval Relation>::= [<Interval-Interval Relation>]<br>(<Time Distance> | ([APPROXIMATELY] (STARTS WITH |<br>| STARTED BY | AT | FINISHES WITH | FINISHED BY | UNTIL |<br>| FOLLOWS)) | INCLUDES | DURING |<br>| OVERLAPS (SINCE) | OVERLAPPED BY (UP TO) )</td></tr>
<tr><td>(R20)</td><td><Interval>::= <Direct Interval> | <Propositional Clause></td></tr>
<tr><td>(R21)</td><td><Direct Interval>::= (<Instant>, <Instant>, <Time Extent>) |<br>| BETWEEN<Instant>AND<Instant> | YESTERDAY | TODAY |...</td></tr>
<tr><td>(R22)</td><td><Operator>::= <Reduction Operator> | <Specification Operator></td></tr>
<tr><td>(R23)</td><td><Reduction Operator>::= MEAN_VALUE | ACCUMULATED_VALUE | ...</td></tr>
<tr><td>(R24)</td><td><Specification Operator>::= MAXIMUM | MINIMUM | LAST | ...</td></tr>
<tr><td>(R25)</td><td><Quantifier>::= [APPROXIMATELY] (ALL | THE_MAJORITY |<br>| BETWEEN_3_AND_5 | A_HALF | ...)</td></tr>
</table>

The first rule establishes that from a propositional clause $PC$ a set of three elements is obtained when the rule is executed: history of signal values $(S(t))$, and degrees of fulfillment of spatial and temporal constraints in the proposition. In these semantic rules, $SF$ stands for "spatial fulfillment" and $TF$ for

**Table 2.** Semantic expressions associated to rules in Table 1. In expressions, $U$ stands for the non-temporal universe of discourse

(R1) <Propositional Clause>: $PC(t) = (S(t), SF_{PC}(t), TF_{PC}(t))$
<Prop. Clause>::= <Proposition> [<Temp. Constr.>]
$SF_{PC}(t) = SF_P(t)$, $TF_{PC}(t) = TC(t) \wedge TF_P(t)$
<Prop. Clause>::= <Prop.><Spatial Rel.><Prop.> [<Temp. Constr.>]
$SF_{PC}(t) = SF_P(t) \wedge \left[\bigvee_{t' \in TF'_P} SR\left(S(t), S'(t')\right) \wedge SF_{P'}(t') \wedge TF_{P'}(t')\right]$
$TF_{PC}(t) = TC(t) \wedge TF_P(t)$
(R2) <Proposition>: $P(t) = (S(t), SF_P(t), TF_P(t))$
<Prop.>::=<Gen. Signal> <Temp. Instant>
$SF_P(t) = SF_{GS}(t), TF_P(t) = TCi(t)$
<Prop.>::=<Operator> <Gen. Signal> [<Val. Constr.>] <Temp. Interval>
with <Operator>::=<Specification Operator>
$SF_P(t) = SF_{GS}(t) \wedge SF_{O_S}(t) \wedge \mu_{VC}(t), TF_P(t) = TCI(t)$
with <Operator>::=<Reduction Operator>
$P(t_{now}) = S_{O_R}(t_{now}) \Rightarrow PC(t_{now}) = (S_{O_R}(t_{now}), SF_{PC}(t_{now}))$
<Prop.>::=<Quantifier> <Gen. Signal> [<Val. Constr.>] <Temp. Interval>
$SF_P(t) = SF_{GS}(t) \wedge \mu_{VC}(t), TF_P(t) = TCI(t)$
(R3) <Gen. Signal>: $GS(t) = (S(t), SF_{GS}(t))$
$SF_{GS}(t) = \mu_{VC}(S(t))$
(R4) <Signal>: $S(t)$, $t \in \tau$
(R5) <Value Constraint>: $\mu_{VC}(u), u \in U$
(R6) <Spatial Relation>: $\mu_{SR}(u, u'), u, u' \in U$
(R7) <Temporal Contraint>: $TR(t)$
(R8) <Temporal Instant>: $TCi(t) = \mu_{i-i}(k) \oplus i(t)$
$TCi(t) = \mu_{i-I}(k) \oplus I(t)$
(R9) <Temporal Interval>: $TCI(t) = \mu_{I-i}(k) \oplus i(t)$
$TCI(t) = \mu_{I-I}(k) \oplus I(t)$
(R10) <Instant-Instant Relation>: $\mu_{i-i}(k), k \in \mathbb{Z}$
(R11) <Instant-Interval Relation>: $\mu_{i-I}(k), k \in \mathbb{Z}$
(R12) <Time Distance>: $\mu_D(k) \oplus \mu_{AFTER/BEFORE}(k)$
(R13) <Time Extent>: $\mu_D(k)$
(R14) <Temporal Quantity>: $k \in \mathbb{Z} \mid \mu_{LITTLE} \mid \ldots$
(R16) <Instant>::=<Direct Instant>: $i(t) = \mu_i(t)$
<Instant>::=<Propositional Clause>: $i(t) = TF_{PC}(t)$
(R17) <Direct Instant>: $t \in \tau \mid \mu_{NOW}(t) \mid \mu_{TODAY}(t) \mid \ldots$
(R18) <Interval-Instant Relation>: $\mu_{I-i}(k), k \in \mathbb{Z}$
(R19) <Interval-Interval Relation>: $\mu_{I-I}(k), k \in \mathbb{Z}$
(R20) <Interval>::=<Direct Interval>: $I(t) = \mu_I(t)$
<Interval>::=<Propositional Clause>: $I(t) = TF_{PC}(t)$
(R21) <Direct Interval>::= $\mu_I(t)$ defined as mentioned in section 2.1
(R23) <Reduction Operator>: $S_{O_R}(t_{now}) = O_R(S(t)), t \in SUPP_{t_{now}}(TCI)$
(R24) <Specification Operator>: $SF_{O_S}(t)$ degree of fulfillment of the specification
(R25) <Quantifier>: any valid quantifier operator $O_Q$ may be used as stated in section 4.1

"temporal fulfillment", whilst the subscripts indicate the entity being referred to.

All time dependent expressions are defined over the time points $t \in \tau$, according to the assumed discretization, except for the case of reduction operators in R2, where a single value of the signal is obtained at $t_{now}$ (R23) when the rule is executed, with its corresponding spatial degree of fulfillment if there exists a value constraint in the proposition acting on this "reduced" spatial value. In the next section an explanation on the meaning of these semantic rules is presented, together with some examples.

## 4 Execution of Fuzzy Temporal Rules

The execution of a rule involves the evaluation of each one of the propositions $PC_m$, $m = 1,...,M$, which make up its antecedent part, and for which an individual DOF will have to be obtained. The calculation of a global DOF for the entire antecedent part, is made by means of the usual conjunction process in fuzzy control: $DOF_{ant\_part} = DOF_1 \wedge DOF_2 \wedge \ldots \wedge DOF_M$. This global value will be the one that is transmitted to the consequent part when the rule is executed.

### 4.1 Execution of Independent Propositions

A proposition $PC_m$ is defined as **independent** if all its associated constraints are absolute, i.e. can be evaluated independently from any other temporal fact or proposition. It corresponds to an instantiation of rule R1 as

<Propositional Clause>::=<Proposition>

and no other recursive instantiation of this rule.

The calculation expression for $DOF_m$ depends on the kind of proposition we are dealing with. If the temporal reference for the proposition is an instant (first case in rule R2, Table 2), we have [4,5]:

$$DOF_m(t_{now}) = \bigvee_{t \in SUPP_{t_{now}}(TF_{PC_m})} SF_{PC_m}(t) \wedge TF_{PC_m}(t) \tag{2}$$

In the case of the temporal reference being an interval (the remaining options in rule R2), we must distinguish the three situations in Table 2: specification operator, reduction operator and quantifier [1].

For the first case (**specification**), the expression for $DOF_m(t_{now})$ will be the same as in (2). In the case of **reduction** operators, the spatial value obtained is not a function of $t$, as we previously mentioned, so the calculation of $DOF_m(t_{now})$ will be as follows (applying also rule R23):

$$\begin{aligned} DOF_m(t_{now}) &= SF_{PC_m}(t_{now}) \\ SF_{PC_m}(t_{now}) &= \mu_{VC_m}(S_{O_R}(t_{now})) \end{aligned} \tag{3}$$

[1] For implementing these operators, respectively denoted as $O_R$, $O_S$ and $O_Q$, any of the valid models in the literature may be used (e.g., [8])

Finally, the structure of the expression for this calculation in the third case (**quantification**) is a function of the particular quantifier in the proposition (R25). In general:

$$DOF_m(t_{now}) = O_{Q_{t \in SUPP_{t_{now}}(TF_{PC_m})}} \left[ SF_{PC_m}(t), TF_{PC_m}(t) \right] \tag{4}$$

where $O_Q$ may be modelled using any valid quantifier model [8]. For instance, when $Q = \forall$:

$$DOF_m(t_{now}) = \bigwedge_{t \in SUPP_{t_{now}}(TF_{PC_m})} \left[ SF_{PC_m}(t) \vee [1 - TF_{PC_m}(t)] \right] \tag{5}$$

and when $Q = \exists$ (this is assumed to be the default case):

$$DOF_m(t_{now}) = \bigvee_{t \in SUPP_{t_{now}}(TF_{PC_m})} \left[ SF_{PC_m}(t) \wedge TF_{PC_m}(t) \right] \tag{6}$$

**Fulfillment of the value constraints.** Different cases may come about, depending on the type of constraints present in the proposition:
(i) The value constraint $VC$ in rule R3 is an absolute value (e.g. $VC$ ="high"). Then, $\mu_{VC}(u) = \mu_{high}(u)$ (R5). This case includes those constraints that are "relative" to an absolute reference value: $\mu_{VC}(u) = (\Delta \oplus V_{ref})(u)$, where $\Delta$ represents the spatial relationship with respect to a reference value $V_{ref}$ ($SR(u, u') = \Delta(u - u')$, e.g., "*The temperature is much greater than 30$^o$C* ").
(ii) If a reduction or a specification operator exists, a second value constraint (rule R2) will be added (with t-norm *minimum*) to the result of applying the first one (in the case this exists).
**Example 1**: "*At some point in the last few minutes, the minimum value among high temperatures has been very high*".
$PC$ : ($GS$ = ($S$ =temperature, $VC_1$ = high), $TCI$=the last few minutes, $O_S$=minimum, $VC_2$=very high).

The value constraint $VC_2$ "*very high*" is applied on the value instance obtained after applying specification operator $O_S$ in the temporal reference $TCI(t)$ "*the last few minutes*" and the value instance of observed temperature values, previously filtered through value constraint $VC_1$ "*high*".

**Fulfillment of the temporal constraints.** The evaluation of the degree of fulfillment or membership to the temporal part of the proposition implies obtaining a distribution $TF_{PC}(t)$, associated to the set of time points on which the rest of the constraints have to be evaluated.
(i) Only temporal constraints $TCI$ or $TCi$ and context $TC$, both absolute temporal references, are present: $TF_{PC}(t)$ will be determined by the possibility distributions associated to these references (either temporal instants or intervals, fuzzy or not), $TF_{PC}(t) = TCI(t) \wedge TC(t)$ (and analogously for the case of instants $TCi$).

(ii) If a temporal constraint $TC$ is relative to another absolute temporal reference $T^*$ (instant or interval) we have (eq. (1)) $TC = D \oplus T^*$, $D$ representing the relationship between temporal entities, and thus, $TF_{PC}(t) = (D \oplus T^*)(t)$.
**Example 2:** "*Temperature was high at some point during the last 30 minutes*".

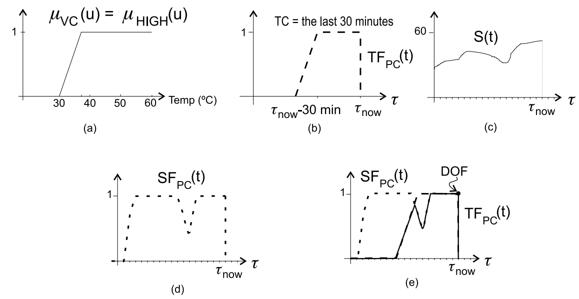


**Fig. 1.** Calculation of DOF for proposition "*Temperature was high at some point during the last 30 minutes*"

We assume that the membership function in Fig. 1(a) represents the value constraint "*high*", time membership function in (b), the time interval "*the last 30 minutes*" (temporal constraint), and the recent history of temperature observed values ($S(t)$) is the one described in (c). The fulfillment of the spatial part $SF_{PC}(t)$ (in (d)) must be combined with the temporal constraint $TF_{PC}(t)$, as shown in (e), in order to calculate the DOF. In this example, we must use the expression for existential quantification (6).

### 4.2 Execution of Dependent Propositions

When in a proposition $PC_m$ constraints that are a function of relative values appear, $PC_m$ can be decomposed into a set of several related propositions (**dependent** propositions), making all possible value or temporal dependences explicit: $PC_m = (P^1, P^2, ..., P^K)$. Thus, in order to obtain $DOF_m$ each one of the constraints in $P^k$, $k = 1,...,K$, has to be evaluated. Dependencies in the propositions can be either spatial (of value) or temporal. Depending on the type of constraint which produces the dependency, this evaluation provides the corresponding value and time instances, and associated DOFs.

**Value Constraints.** In dependent propositions, the reference value makes reference to the value instance obtained after another proposition has been evaluated. This corresponds to an instantiation of rule R1 in Table 1 as:
<Propositional Clause>::= <Proposition><Spatial Relation><Proposition>.
Here, in order to evaluate $PC_m$ two propositions (denoted as $P^k$ and $P^{k'}$) should be evaluated.

(i) In the case of relative value constraints,

$$SF_{P^k}(t) = \bigwedge_{t^{k'} \in TC^{k'}} \mu_{SR}(S^k(t), S^{k'}(t')) \wedge SF_{P^{k'}}(t')$$

$SF_{P^{k'}}$ being the value instance obtained after evaluating the proposition $P^{k'}$ being referred to ($SF_{P^{k'}}$ can be provided by the evaluation of a value constraint $VC^{k'}$, of a specification operator...)
(ii) Whenever a reduction or specification operator exists, the degree of spatial fulfillment $SF_{P^k}$ corresponds to the compatibility between the resultant of applying operator $O^k$ to the corresponding value instance of proposition $P^k$ over the temporal reference, and the value constraint given by $SR$ applied on the value instance of the dependent proposition $P^{k'}$.
**Example 3**: "*Throughout a minute the mean value of temperature in heater 1 is greater than the mean value of temperature in heater 2*". According to the proposed model of propositions, $PC = (P^1, SR, P^2, TC)$:
$P^1$ : ($S^1$=temperature in heater 1, $O_R^1$=mean value), $SR$=greater than
$P^2$ : ($S^2$=temperature in heater 2, $O_R^2$=mean value)
$TC$=$TCI$=($ib, ie, D$=1 minute).

The evaluation of both propositions is linked through the references in the specification of $P^1$ with respect to $P^2$, by means of a spatial relation $SR$. Therefore, $P^2$ must initially be evaluated: over the temporal context (in the case of it existing) or over all the history of values, temporal instances will be taken (intervals in this case, 1 minute long). In each one of these instances, the operator $O_R^2$ will be evaluated, providing the mean value, in that minute, of all the temperature values of heater 2. In order to evaluate proposition $P^1$, we will obtain from $P^2$ the set of temporal instances (intervals of 1 minute) and the associated value instances (for each interval 1 minute long, the mean value of temperature in heater 2). This set will be the reference set for the possible evaluation instances for $P^1$. From each element, the temporal part will be used to select values of temperature 1, in order to subsequently obtain the mean value (constraint $O_R^1$), and then the constraint given by the spatial relation $SR$ (mean value of temp. 1 "greater than" mean value of temp. 2) will be evaluated. In this way, we obtain a set of values $SF_{PC}$ for each evaluation instance of $P^1$. The spatial fulfillment for $PC(SF_{PC})$ will be obtained from the best of the possible instances: the one that verifies all the constraints to the highest degree.

**Temporal Constraints.** We say that a dependent proposition through temporal constraints exists when in a proposition $P^k$ a reference is made to the temporal instance that has been calculated for another proposition $P^{k'}$.
(i) The reference value is given by the time of occurrence of a fact. This corresponds to instantiation of rules (R8) or (R9) giving a temporal relation to an <Instant> or an <Interval>, and then:
(R16) <Instant>::= <Propositional Clause>

(R20) <Interval>::= <Propositional Clause>
In this case, the temporal reference (instant or interval) in a proposition $P^k$ is given by a reference to a proposition $P^{k'}$ describing the occurrence of the considered fact, so this second proposition must initially be solved. Evaluating $P^{k'}$ we obtain the spatial values verifying the fact, $SF_{P^{k'}}$ (degree of fulfillment of this fact), and the set of time points verifying the constraint with a non-null value ($TF_{P^{k'}}$). Therefore, we have that $TF_{P^k}(t) = (D \oplus TF_{P^{k'}})(t)$.

**Example 4**: *"Pressure was high a little before temperature was low at some point during the last half an hour"*. $PC = (P^1, TC)$
$P^1 : (GS^1 = (S^1\text{=pressure}, VC^1\text{=high}), TCI^1\text{=a little before}(P^2), Q^1{=}\exists)$
$P^2 : (GS^2 = (S^2\text{=temperature}, VC^2\text{=low}), TCi^2{=}t)$
$TC$=the last half an hour.

The DOF of proposition $P^1$ depends on the evaluation instance of proposition $P^2$. Figure 2(a) shows the history of recent temperature values $S^2(t)$, assuming $\delta = 5min$. These values are matched with the value constraint "*low*" in order to obtain $SF_{P^2}$, shown as *Temp. low* in Fig. 2(b), where the representation of the temporal context $TC$ "*the last half an hour*" is also given. $TF_{P^2}$ represents the temporal instance obtained after the evaluation of $P^2$. It is the result of considering both the membership to the temporal context "*the last half an hour*" and the value constraint "*low*". Furthermore, it provides the temporal reference for the evaluation of $P^1$. Within the temporal context $TC$ we have different temporal instances in which the value constraint has to be evaluated, thus we obtain a $SF_{P^2}$ for each one: how low the temperature is.

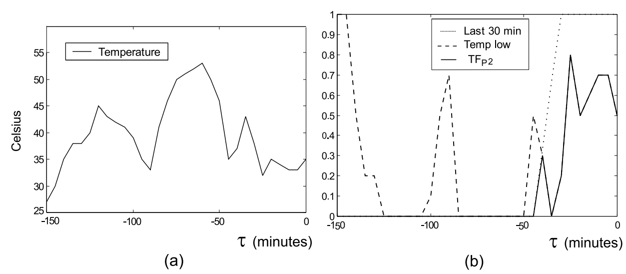


**Fig. 2.** Calculation of the temporal instance for proposition $P^2$ in example 4. In the time axis, we are assuming $t_{now}$ as the time origin ($t = 0$), in order to make the representation more simple

The process of calculating the fulfillment of the temporal constraint $TCI^1$ is shown graphically in Fig. 3. The temporal relationship "*a little before*" is assumed to be defined as the possibility distribution shown in Fig. 3 (a). The temporal reference $TCI^1$ is obtained as the fuzzy addition between this possibility distribution and the temporal instance $TF_{P^2}$, as shown in Fig. 3 (b).

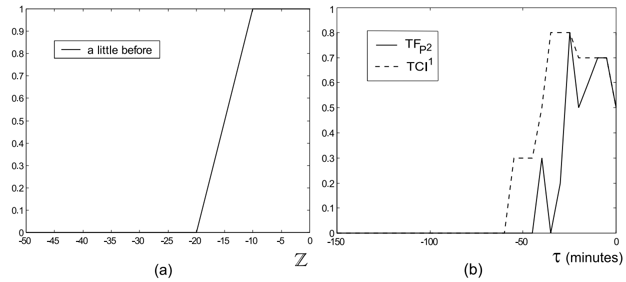


**Fig. 3.** Calculation of the temporal reference for proposition $P^1$ in example 4: "*a little before* $(TF_{P^2})$"

(ii) Finally, we can also have temporal constraints on any operator. In the following example we can see both cases (specification operator and temporal constraint on the selected value):

**Example 5**: *"Between 6 and 8, pressure was high a little after the last time temperature was high"*. $PC = (P^1)$

$P^1$:($GS^1$=($S^1$=pressure, $VC^1$=high), $TCI^1$=a little after $P^2$)

$P^2$:($GS^2$=($S^2$=temperature, $VC^2$=high), $O_S^2$=last, $TC^2$=between 6 and 8).

$P^2$ will be the first to be evaluated, providing $P^2(t)$ ($SF_{P^2}(t)$ and $TF_{P^2}(t)$), with $t \in SUPP_{TF_{P^2}}$ being the time points in $TC^2$ verifying $SF_{VC^2}(t) \neq 0$. Thus, $TCI^1$ will be obtained as: $TCI^1(t) = (D \oplus TF_{P^2})(t)$.

## 5 Discussion

The generalization of fuzzy rules to FTRs, which allows an explicit representation and handling of time, can, no doubt, contribute to extending the application domain of fuzzy logic, or at least, to making the design of solutions for problems associated to dynamic systems and processes easier. In this sense, the control environment is a paradigmatic one, although, paradoxically, FTRs are not yet being used in it as we think they should.

In this work we introduce a model of FTRs that attempts to represent the semantic expressiveness of expert knowledge. The expressive capacity of the model and how it directly deals with imprecision and uncertainty linked to information and knowledge, make it, in our opinion, very promising for application in environments such as process control [13] or monitoring.

Regarding our current and future work, our aim is to extend the language, in such a way that definitions for other temporal facts are included and also other cases of interest in fuzzy temporal reasoning systems are considered.

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