A Model of Fuzzy Temporal Rules for Knowledge Representation and Reasoning

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Abstract

In this paper a model for the representation and execution of fuzzy temporal rules is presented. The model here described permits using temporal references in the propositions either absolute or relative to occurrence of events. A complete formal description for the representation model is proposed, together with examples. For the reasoning model, some expresions for common use situations are also presented.

Keywords: fuzzy temporal rule, temporal reasoning

1 Introduction

The control environment is essentially an application field in which variables are dynamic and changeable. In spite of this, it is curious to note that there has been a tendency, it would seem, to opt for simple and basic fuzzy reasoning models, without paying hardly any attention to the possibility of representations of knowledge that explicitly manipulate time and reason on it. In our opinion, this noticeably limits the possibilities for the expansion of fuzzy reasoning to applications which demand more expressive forms of dealing with expert knowledge. The greater complexity required at an operational level by fuzzy temporal rules (FTRs) may be, without doubt, one of the causes of this. On the other hand, the fact that there are still no well-defined models nor computer aid tools for the design of FTR-based systems, means that the use of this form of knowledge representation is very limited, which, in turn, limits the interest of the research and technical communities in FTRs.

In this work, our objective is to present an initial approach to a formal FTR model, which may be used for representation of and reasoning on fuzzy temporal knowledge. We present an FTR model that is endowed with substantial expressive capabilities, and analyze the FTR execution process, including some examples by way of illustration.

Vitek [13] was the first to attempt to formalize the representation of fuzzy time by means of intervals associated to fuzzy sets. Dubois and Prade formalize the concept of fuzzy time in [6], laying the foundations for the representation of imprecision and uncertainty in temporal knowledge with the theory of possibility. They introduce the concepts of *date* and *fuzzy time interval*, shaping the possible fuzzy relations between these temporal entities through *fuzzy relations on the real line.*

With regard to models of fuzzy temporal reasoning (fuzzy reasoning that explicitly takes into account the presence of temporal references), these fundamentally deal with constraint network models [2, 9] and fuzzy rules which, in one way or another, explicitly introduce time as another decision variable. In [10] Qian introduces a model for what he calls fuzzy proposition reasoning and temporal reasoning, defining a temporal descriptor $TD_i(P_i)$ as an operator describing the time characteristics of a fuzzy proposition P_i. The temporal descriptor can describe three forms of temporal relationship: absolute ("real clock" time description), relative (related to a temporal reference point), and before/after (particular case of a relative time description). One of the main, and most important, limitations of Qian's model is that it is incapable of handling persistence over time of conditions of the value of the variables that are handled. On the contrary, one very relevant characteristic of this model is that it enables us to deal with all basic temporal relations [1] amongst temporal entities (time instants or intervals).

Another proposal for FTRs was given by Virant and Zimic [12], where one of the most interesting contributions is the introduction of the concept of "time dependent fuzzy set" ($A(t)$). Furthermore, they introduce the concept of "fuzzy time operator" (D^k) and its application to fuzzy sets. Maeda *et al.* [8] propose a reasoning method that incorporates a vague time delay into fuzzy *if-then* rules (dynamical fuzzy reasoning method). This is a similar proposal to that of Carse *et al.* [5], which deals with establishing a validity time for the result inferred by a rule. Raha and Ray [11] put forward a model for the representation and manipulation of vague time, following the scheme of [6]. As an innovative aspect, they indicate that on occasion, it is necessary to represent temporal entities by using relational matrices.

2 Model of Fuzzy Temporal Rules

2.1 Time Ontology

For representing temporal entities, and following [6, 7], we assume a discrete time axis τ, where time point $t_0 \in \tau$ is assumed to be the time origin, and $\forall k \in \mathbb{Z}$ ($\mathbb{Z}$ being the set of integer numbers), $\delta = t_k - t_{k-1}$ is assumed to be a constant. The following temporal entities [2] are also considered:
- **Instant** i. Represented as a normalized and unimodal possibility distribution $\mu_i(t_k)$: represents the possibility of i being precisely time point $t_k \in \tau$.
- **Temporal distance (duration)** D. Represents quantities of time. A fuzzy temporal distance is represented by a possibility distribution μ_D over $\mathbb{Z}$, the elements of $\mathbb{Z}$ representing units of time δ.
- **Interval** $I(i_b, i_e, D)$. Defined based on its beginning i_b and ending i_e instants, and its duration D. In general, it is managed by means of a possibility distribution μ_I over τ comprising the time points that are possibly after i_b and before i_e.

Furthermore, we consider the basic temporal relationships (qualitative and quantitative) [2] between these entities, at the level of instants and intervals ($i-i$, $i-I$, $I-I$), and between temporal distances ($D-D$). We assume that all these relationships can be reduced to relationships between time points and temporal distances, where the function that defines this relationship can be expressed as [6]:

$$R(t, t') = \Delta(t - t') = \Delta(k\delta), k \in \mathbb{Z} \qquad (1)$$

Some examples of this kind of relationships are: *before, at the end of...* We do not consider in any case expressions that make reference to the future.

Using operator $\oplus$, that represents the fuzzy addition operation, defined as:

$$\forall t \in \tau, \mu_{D\oplus T}(t) = Sup_{t=t'+m\delta} min\left(\mu_D(m), \mu_T(t')\right),$$

with $t' \in \tau, m \in \mathbb{Z}$, we have

* **Absolute time**: T, which can always be expressed as $D \oplus t_0$.
* **Relative time**: $D \oplus T_{ref}$, bearing in mind that it always has to be prior (or equal) to t_{now} (the current time point). T_{ref} is in this case the entity that is associated to the occurrence of the reference fact.

In this manner, we may have absolute and relative temporal entities. We should strictly consider that the specification of time associated to a fact in absolute terms is any one relative to a fixed entity of time, whilst a specification of time associated to a fact in relative terms will be the one that depends on a temporal entity that is linked to the occurrence of other facts.

2.2 Ontology of facts

We assume that we are operating with discrete signals S, described by means of a function $S(t)$, which represents the history of crisp values.

We suppose that there are two types of signals:
- **Observable**: signals whose values are supplied from outside the system, by means of sensors, file readings, etc.
- **Inferible**: signals whose values are supplied by the system itself, by means of inference processes based on prior observations and/or inferences.

We also assume two basic types of facts that can be referred to in the FTRs:
- **Event**: a fact associated to a temporal instant, fuzzy or not (it may be a simple numeric or symbolic value). It is a fact with a null duration (its

real duration is inferior to the temporal unit).
- **Episodes**: a fact associated to a temporal interval, fuzzy or not, in which the conditions which identify the fact itself persist. It is a temporal fact with a non null duration.

All the facts (observed or inferred) are associated to absolute times, precise or not. This allows the establishment of a total order between the known facts. On the other hand, it will be assumed that the signal histories will conform sequentially in time. We are therefore orientating our model towards real time applications.

2.3 Model of Fuzzy Temporal Rules and Propositions

FTRs will take the form:

IF P_1 and P_2 and ... and P_M THEN C_1 and C_2 and ... and C_N

where P_m, $m = 1, \ldots, M$, are propositions of the antecedent part of the rule, and C_n, $n = 1, \ldots, N$, of the consequent part of the rule (conclusions), which take the form $C_n : (S_n, V_n, T_n)$, S_n being an inferible signal, V_n the value (represented by a non-temporal fuzzy set) inferred for this signal and T_n the time (fuzzy instant or interval) associated to the inferred value.

For the antecedent part, we contemplate a proposition format with a great degree of expressiveness, which enables us to represent facts in which information of a spatial or a temporal type that is linked to them may be fuzzy, and be given in a manner that is absolute or relative to other facts.

The most general form of the **fuzzy temporal proposition** is the following one:

$P : (S, C_V, C_T, TC, O, C_{VO}, C_{TO})$,

where S is a signal and the remaining elements, different spatial or temporal constraints on it:

- **Value Constraints:** C_V is a spatial (non temporal) value constraint on the signal. It may be given in an absolute manner ($C_V(u) = V(u)$, e.g. "*high*", V representing the possibility distribution that defines linguistic value *high*), or related to another spatial reference value V_{ref} ($C_V(u) = (\Delta \oplus V_{ref})(u)$, e.g. "*greater than 30º*"). In the latter case, it is supposed that any constraint based on a relation between spatial values will take this form, $\Delta(u - u')$ being the function that defines the relationship between spatial values on the universe U and $\oplus$, the operator of the fuzzy addition.

- **Temporal Constraints:** C_T is an absolute or relative temporal constraint. Examples of this type of constraint are "*throughout the last half an hour*", "*before the maximum value of temperature*".

- **Temporal Context:** TC is the temporal context of signal evaluation, which establishes a temporal window (interval) within which the proposition will be evaluated.

- **Operators:** O represents an operator belonging to one of the following types:

* **Quantification operators**: *all, the majority, between 3 and 5, approximately half,...* In some cases it may be of interest to quantify the fuzzy temporal propositions, as in "*the temperature has been high during the majority of the last half an hour*".

* **Specification operators**: *first, last, maximum, minimum,...* These select one candidate from amongst various, according to a specific criterion (spatial and/or temporal).

* **Reduction operators**: *mean value, accumulated value,...* In this case the constraint operates on the spatial values that are observed or inferred for the proposition, in order to return a new one, calculated on the basis of the former ones.

- **Second order value constraints**: C_{VO} is a spatial value constraint, which acts on the resultant of applying an operator O ("*greater than* the mean value"...).

- **Second order temporal constraints**: C_{TO} is a temporal constraint, acting on the resultant of an operator O ("*a little after* maximum value"...).

The following examples show representations of different propositions in accordance with the model we propose:

Example 1: *"The majority of the temperature values throughout the last few seconds have been high"*, is represented by:
P : (S=temperature, C_V=high, C_T=the last few seconds, O_Q=the majority).

Example 2: *"The mean value for the temperature over the last 48 hours was moderate"*.
P : (S=temperature, C_T=last 48 hours, O_R=mean value, C_{VO} =moderate).

3 Execution of Fuzzy Temporal Rules

In principle, for each one of the constraints present in a proposition we can obtain an associated degree of fulfillment (DOF), which indicates the degree in which this constraint is verified according to the *evaluation instance*, EI, understood as the set of data (spatial and temporal values and, where appropriate, associated DOFs) considered in order to be compared with the proposition at the instant of the evaluation of the rule. The evaluation time for a rule will be the current time point t_{now}, which may be used as a reference time in the propositions in the antecedent part of the rule. On the other hand, it will represent the reference instant for the consequent part of the rule, in the case of the rule being executed. Furthermore, it should be borne in mind that during the evaluation of propositions in real time, the evaluation scenario will alter as time advances. In the case of there being a temporal context TC defined explicitly for the proposition, different evaluation instances may be obtained from it, so that the remaining constraints will be evaluated on each one of them. In the absence of other explicit criteria, it will be considered that the result of the evaluation of a proposition in a given temporal context is the one that is obtained on the basis of the best of the possible evaluation instances. All these partial DOFs contribute to obtaining a DOF_m for each proposition P_m. The execution of a rule involves the evaluation of each one of the propositions P_m, $m = 1, \ldots, M$, which make up its antecedent part, and the calculation of a global DOF for the entire antecedent part, by means of the usual conjunction process in fuzzy control.

3.1 Execution of independent propositions

A proposition P_m is defined as **independent** if all its associated constraints are absolute, i.e. can be evaluated independently from any other temporal fact or proposition.

The DOFs of the spatial and temporal parts of the proposition participate in the calculation of DOF_m, in the form shown in [3, 4]. The structure of the expressions for this calculation is a function of the existence or not of quantifiers in the proposition, and of their type. In general, $DOF_m = f_{O_Q}(V_m, T_m)$, where V_m represents the degree of fulfillment of the value constraints, and T_m, the temporal distribution induced by the temporal constraints. O_Q is the particular quantification operator (in the case of it existing).

3.1.1 Fulfillment of the value constraints

For independent propositions, the degree of "spatial fulfillment" (V_m) is calculated considering the different value constraints on the signal:

$$V_m(t) = f\left(DOF_{C_{V_m}}(t), O_m(t), DOF_{C_{VO_m}}(t)\right)$$

Different cases may come about, depending on the type of constraints present in the proposition:

(i) The value constraint C_{V_m} is an absolute value: $C_{V_m}(u) = V(u)$ (e.g. V ="high"):

$$DOF_{C_{V_m}}(t) = \bigvee_{u \in U} IV(t,u) \wedge V(u)$$

where IV represents observed or inferred values for the signal, and U the corresponding universe of discourse. This case includes those constraints that are "relative" to an absolute reference value: $C_{V_m}(u) = (\Delta \oplus V_{ref})(u)$, where Δ represents the spatial relationship R with respect to a reference value V_{ref} ($R(u,u') = \Delta(u-u')$, for example, "*The temperature is much greater than 30^{o}C* ").

(ii) If a reduction or a specification operator O exists, a second order constraint C_{VO_m} will operate in the form:

$$V_m(t) = \bigvee_{u \in U} O_{t' \in SUP(T_m)}[IV(t',u), T_m(t')] \wedge V(u)$$

V being the value associated to the value constraint ($C_{VO_m}(u) = V(u)$), and $T_m(t)$ the temporal reference induced by the temporal constraints present in the proposition; the operator O acts on the value instance of the signal IV and the temporal entity T_m and selects or obtains a new value instance, on which the corresponding constraint C_{VO_m} is applied. This can be seen in the following example:

Example 3: "*The maximum value of temperature in the last few minutes has been high*".
P : (S =temperature, C_T=the last few minutes, O_S=maximum, C_{VO}=high).

The value constraint "*high*" is applied on the value instance IV^*, obtained after applying operator O_S to the reference $T(t)$ defined by the temporal constraint "*the last few minutes*" and the value instance IV of observed temperature values.

3.1.2 Fulfillment of the temporal constraints

The evaluation of the degree of fulfillment or membership to the temporal part of the proposition implies obtaining a distribution $T_m(t)$, associated to the set of time points on which the rest of the constraints have to be evaluated. Hence, the most general case will be a proposition with temporal constraints, context and operators:

$$T_m(t) = C_{T_m}(t) \wedge TC_m(t) \wedge DOF_{O_m}(t) \wedge C_{TO_m}(t)$$

Nevertheless, it is worthwhile describing some of the more simple situations in a more detailed way:
(i) Only temporal constraints C_{T_m} and context TC_m, both absolute temporal references: $T_m(t)$ will be determined by the possibility distributions associated to these references (either temporal instants or intervals, fuzzy or not),

$$T_m(t) = C_{T_m}(t) \wedge TC_m(t)$$

(ii) If the temporal constraint C_{T_m} is relative to another absolute temporal reference T^* (instant or interval) we have (sect. 2.1) $C_{T_m} = D \oplus T^*$, D representing the relationship between temporal entities, and thus,

$$T_m(t) = (D \oplus T^*)(t) \wedge TC_m(t)$$

which, as can be seen, is reduced to the previous case, as happened with spatial constraints.
(iii) When operators O_m which act on the temporal part of the proposition exist (e.g. "*last*"):

$$T_m(t) = DOF_{O_m}(t) \wedge C_{T_m}(t) \wedge TC_m(t)$$

If in example 3 the specification operator is changed to "*last*": "*The last value of temperature in the last few minutes has been high*", we have:

$$\begin{aligned} V_m(t) &= \bigvee_{u \in U} O_{t' \in SUP(T)} \left(IV(t', u), C_T(t')\right) \wedge V(u) \\ T_m(t) &= DOF_O(t) \wedge C_T(t) \end{aligned}$$

where S="*temperature*", C_T="*the last few minutes*", O="*last*" and C_{VO}=V="*high*". We see that when O is applied on the value instance IV of the signal, within the temporal reference given by C_T, a spatial value is obtained (on which the constraint C_{VO} will be applied), as well as a DOF_O (degree of fulfillment of the characteristics defining the operator).

Example 4: "*Temperature was high at some point during the last 30 minutes*", is described as:
P : (S=Temperature, C_V=high, C_T=the last 30 minutes, O_Q=$\exists$).

We assume that the membership function in

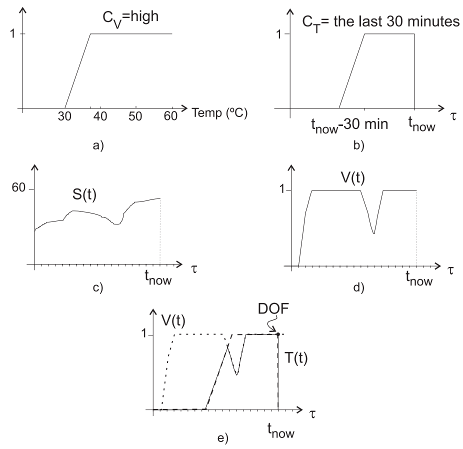

Figure 1: Calculation of DOF for proposition "*Temperature was high at some point during the last 30 minutes*".

Fig. 1(a) represents the value constraint "*high*", time membership function in (b), the time interval "*the last 30 minutes*" (temporal constraint), and the recent history of temperature observed values ($S(t)$) is the one described in (c). To obtain the DOF of the proposition, we must combine the fulfillment of the spatial part $V(t)$ (in (d)) with the temporal constraints, as shown in (e).

3.2 Execution of dependent propositions

When in a "natural language" proposition P_m constraints that are function of relative values appear,

P_m is decomposed into a conjunction of several related propositions (**dependent** propositions), making all possible value or temporal dependences explicit: $P_m = P^1 \wedge P^2 \wedge \ldots \wedge P^K$. Thus, in order to obtain the DOF for P_m, each one of the constraints in P^k, $k = 1, \ldots, K$, has to be evaluated. Dependencies in the propositions can be either spatial (of value) or temporal. Depending on the type of constraint which produces the dependency, this evaluation provides the corresponding value and time instances, and associated DOFs.

3.2.1 Value Constraints

In dependent propositions, the reference value makes reference to the value instance $IV^{k'}$ obtained after another proposition $P^{k'}$ has been evaluated.

(i) In the case of relative value constraints, $C_V^k = \Delta \oplus IV^{k'}$, $IV^{k'}$ being the value instance obtained after evaluating the proposition $P^{k'}$ being referred to ($IV^{k'}$ can be provided by the evaluation of a value constraint $C_V^{k'}$, of a selection operator...):

$$V^k(t) = \bigvee_{(u,u') \in U \times U} IV^k(t,u) \wedge (\Delta \oplus IV^{k'})(t,u')$$

Example 5: "*Temperature in heater 1 has been greater than temperature in heater 2 during the last few minutes*". According to our model, this proposition is represented as $P_5 {=} P^1 \wedge P^2$:
P^1 : (S^1=temperature in heater 1, C_V^1=greater than IV^2, $C_T^1 {=} IT^2$) AND
P^2 : (S^2=temperature in heater 2, TC^2=the last few minutes).

Evaluation of P^2 provides the value instance IV^2 (temperature values in heater 2) and IT^2 (time points associated to each of the values). In this way, we can obtain IV^1 (temperature values in heater 1, at each one of the time points in IT^2). On this IV^1, the value constraint C_V^1 ("greater than IV^2") will be applied. Here, $k = 1, k' = 2$.

(ii) If a reduction or specification operator O exists, we have:

$$V^k(t) = \bigvee_{(u,u') \in U \times U} O^k_{t' \in SUP(T^k)}(IV^k(t',u), T^k(t')) \wedge \wedge \left(\Delta \oplus IV^{k'}(t,u')\right)$$

The degree of spatial fulfillment V^k corresponds to the compatibility between the resultant of applying operator O^k to the corresponding value instance of proposition P^k over the temporal reference (T^k), and the value constraint applied on the value instance of the dependent proposition $P^{k'}$.

Example 6: "*Throughout a minute the mean value of temperature in heater 1 is greater than the mean value of temperature in heater 2*". According to the proposed model of propositions, $P_6 = P^1 \wedge P^2$:
P^1 : (S^1=temperature in heater 1, $C_T^1 {=} IT^2$, O_R^1=mean value, C_{VO}^1=greater than IV^2) AND
P^2 : (S^2=temperature in heater 2, $C_T^2 {=} I(ib, ie, D{=}1$ minute), O_R^2=mean value).

The evaluation of both propositions is linked through the references in the specification of P^1 with respect to P^2. Therefore, P^2 must initially be evaluated: over the temporal context (in the case of it existing) or over all the history of values, temporal instances will be taken (intervals in this case, 1 minute long). In each one of these instances, the operator O_R^2 will be evaluated, providing the mean value, in that minute, of all the temperature values of heater 2. In order to evaluate proposition P^1, we will obtain from P^2 the set of temporal instances (intervals of 1 minute) and the associated value instances (for each interval 1 minute long, the mean value of temperature in heater 2). This set will be the reference set for the possible evaluation instances for P^1. From each element the temporal part will be used to select values of temperature 1, in order to subsequently obtain the mean value (constraint O_R^1), and the constraint C_{VO}^1 (mean value of 1 greater than mean value of 2) will be evaluated. In this way, we obtain a set of values $DOF_{C_{VO}^1}$ for each evaluation instance of P^1. The DOF for P^1 will be obtained from the best of the possible instances: the one that verifies all the constraints to the highest degree.

3.2.2 Temporal Constraints

We say that a dependent proposition through temporal constraints exists when in a proposition P^k a reference is made to the temporal instance that has been calculated for another proposition $P^{k'}$.

(i) The reference value is given by the time of occurrence of a fact. In this case, the reference to proposition $P^{k'}$ describing the occurrence of the considered fact must initially be solved. Evaluating $P^{k'}$ we obtain $IV^{k'}$ (spatial values verifying the fact), $DOF_{C_V^{k'}}$ (degree of fulfillment of this fact), $IT^{k'}$ (time points verifying the constraint with a non-null value), and $T^{k'}$ (degree of fulfillment, for the points in $IT^{k'}$, of the temporal constraint). Therefore, we have that $T^k(t) = (D \oplus T^{k'})(t)$, for $t \in IT^{k'}$.

Example 7: *"Pressure was high a little before temperature was low at some point during the last half an hour".* $P_7 = P^1 \wedge P^2$:
P^1 : (S^1=pressure, C_V^1=high, C_T^1=a little before(IT^2), O_Q=∃) AND
P^2 : (S^2=temperature, C_V^2=low, TC^2=the last half an hour).

The DOF of proposition P^1 depends on the evaluation instance of proposition P^2. Figure 2(a) shows the history of recent temperature values $S^2(t)$, assuming $\delta = 5min$. These values are matched with the value constraint "*low*" in order to obtain $DOF_{C_V^2}$, shown as *Temp. low* in Fig. 2(b), where the representation of the temporal context "*the last half an hour*" is also given. IT^2 represents the temporal instance obtained after the evaluation of P^2. It is the result of considering both the membership to the temporal context "*the last half an hour*" and the value constraint "*low*". Furthermore, IT^2 provides the temporal reference for the evaluation of P^1. Within the temporal context TC^2 we have different temporal instances in which the value constraint has to be evaluated, thus we obtain a $DOF_{C_V^2}$ for each one: how low the temperature is.

The process of calculating the fulfillment of the temporal constraint C_T^1 is shown graphically in Fig. 3. The temporal relationship "*a little before*" is assumed to be defined as the possibility distribution shown in Fig. 3 (a). The temporal reference C_T^1 is obtained as the fuzzy addition between this possibility distribution and the temporal instance IT^2, which is shown in Fig. 3 (b).

(ii) If there are also temporal selection operators in the dependent proposition, the DOF of the temporal selection will have to be aggregated to the corresponding degree of temporal fulfillment.

$$T^k(t) = T^{k'}(t) \wedge TC^k(t) \wedge DOF_{O^k}(t)$$

(iii) Finally, we can also have temporal constraints on any operator:

$$T^k(t) = C_{TO}^k(t) \wedge TC^k(t) \wedge DOF_{O^k}(t)$$

In the following example we can see both cases (selection operator and temporal constraint on the selected value):

Example 8: *"Between 6 and 8, pressure was high a little after the last time temperature was high".* $P_8 = P^1 \wedge P^2$:
P^1 : (S^1=pressure, C_V^1=high, C_T^1=a little after(IT^2)) AND
P^2 : (S^2=temperature, C_V^2=high, TC^2=between 6 and 8, O_S^2=last).

P^2 will be the first to be evaluated, providing $IV^2(t)$, $DOF_{C_V^2}(t)$, $DOF_{O_S^2}(t)$ and $TC^2(t)$, with $t \in IT^2$ (set of time points in TC^2 verifying $DOF_{C_V^2}(t) \neq 0$). Thus, T^1 will be obtained as: $T^1(t) = C_{TO}^1(t) \wedge TC^2(t) \wedge DOF_{O_S^2}(t) = (D \oplus IT^2)(t) \wedge TC^2(t) \wedge DOF_{O_S^2}(t)$.

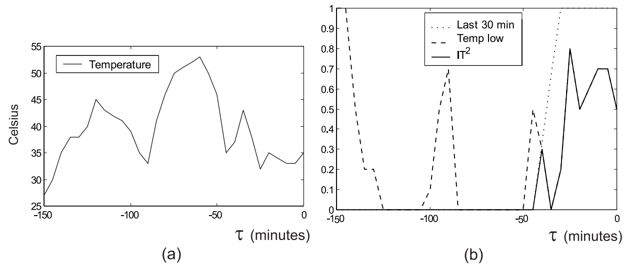

Figure 2: Calculation of the temporal instance for proposition P^2 in example 7. In the time axis, we are assuming t_{now} as the time origin ($t = 0$), in order to make the representation more simple.

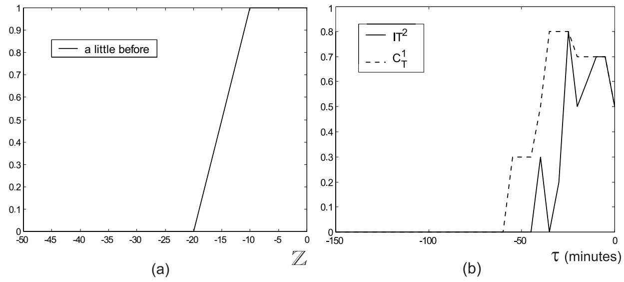

Figure 3: Calculation of the temporal reference for proposition P^1 in Ex. 7: "*a little before (IT^2)*".

4 Discussion

The generalization of fuzzy rules to FTRs, which allows an explicit representation and handling of time, can, no doubt, contribute to extending the application domain of fuzzy logic, or at least, to making the design of solutions for problems associated to dynamic systems and processes easier. In this sense, the control environment is a paradigmatic one, although, paradoxically, FTRs are not yet being used in it as we think they should.

In this work we introduce a model of FTRs that, although it is incomplete, attempts to represent the semantic expressiveness of expert knowledge. The expressive capacity of the model and how it directly deals with imprecision and uncertainty linked to information and knowledge, make it, in our opinion, very promising for application in environments such as process control or monitoring.

Regarding our current and future work, our aim is to completely formalize the model, in such a way that all possible interesting cases in fuzzy temporal reasoning systems are considered.

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